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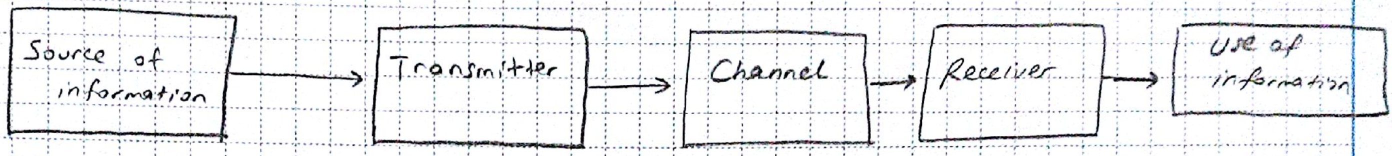
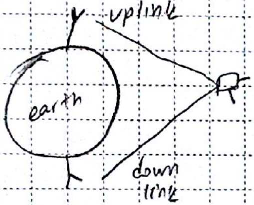


Application of comminication system

1) Broadcasting (Radio, Television)
2) Point to Point (sattalite, wireless com.)



3) Communication Network

Baseband Comminication (işareti değiştirmeden gönderiyoruz.)

Carrier Comminication (Büyük taşıyıcı ile gönderiyoruz)

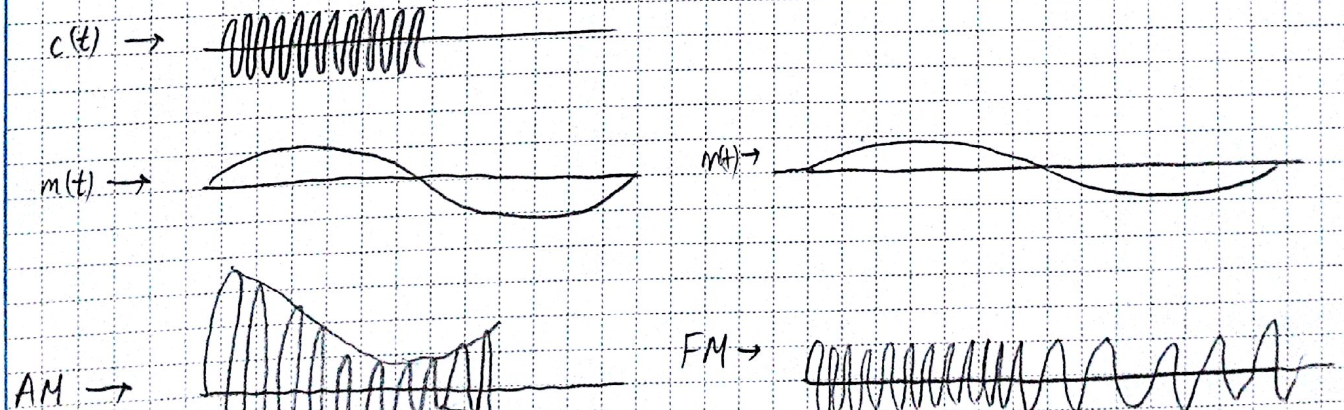
$c = \lambda \cdot f$ (f'i ayarlayarak λ'yı (dalga boyu) onu ayarlayabiliriz.)

Carrier → $c(t) = A\cos(2\pi f c t + \phi)$

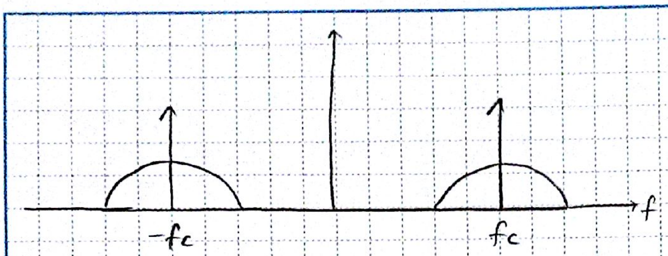
Message → $m(t)$

Modulation Techniques

1) Continous Wave (CW) Modulation

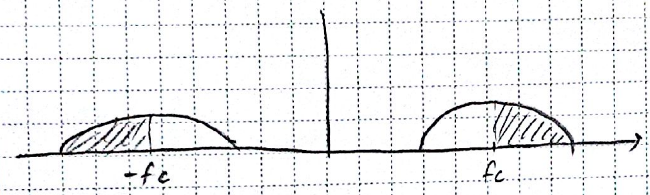


AM

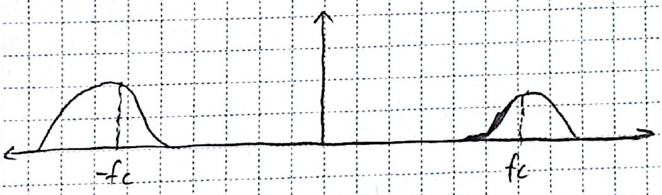


DSB-SC

SSB



VSB



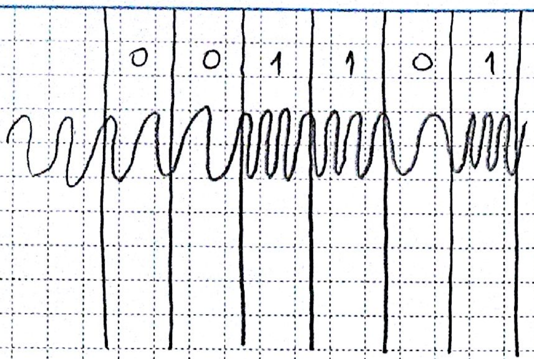
$A \cos(2\pi f_c t + \Phi)$

2) Digital CW Modulation

input binary signal

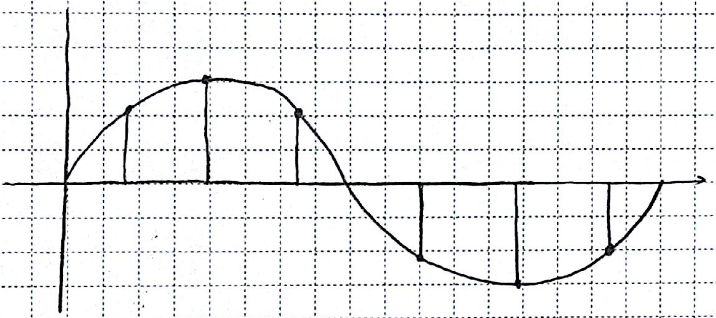
Binary Amplitude-Shift Keying (BASK)

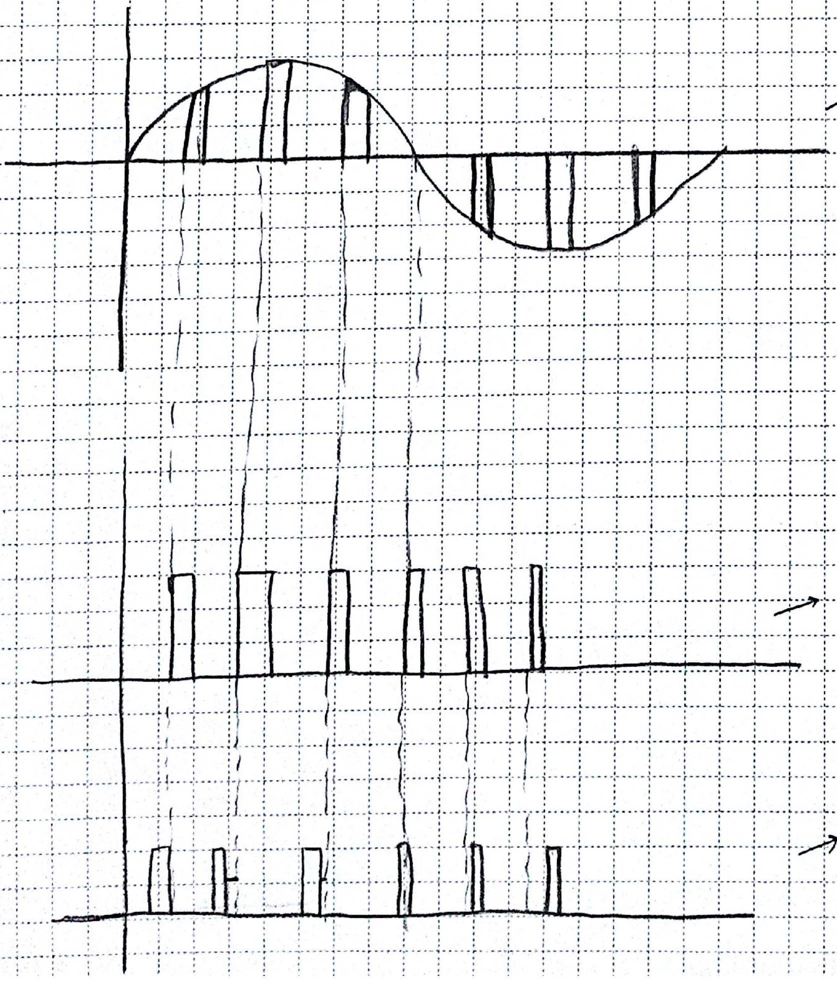
Binary Phase Shift Keying (BPSK)



→ Binary Frequency Shift Keying (BFSK)

3) Pulse Modulation Taşıyıcı olmadan yapılan modulasyon (Baseband)





→ Pulse Amplitude Modulation (PAM)

→ Pulse With modulation (PWM)
Pulse Duration Modulation (PDM)

→ Pulse Position Modulation (PPM)

- Signal and Systems review
- Sampling theory
- Pulse Modulation (PAM, PWM, PPM)
- Source codding (Pulse Code modulation, Delta Modulation)
- Digital Modulation (Frequency ; Phase; Amplitude ; shift Keying)

Midterm	%30
Lab	%15
Hw	%15
Final	%40

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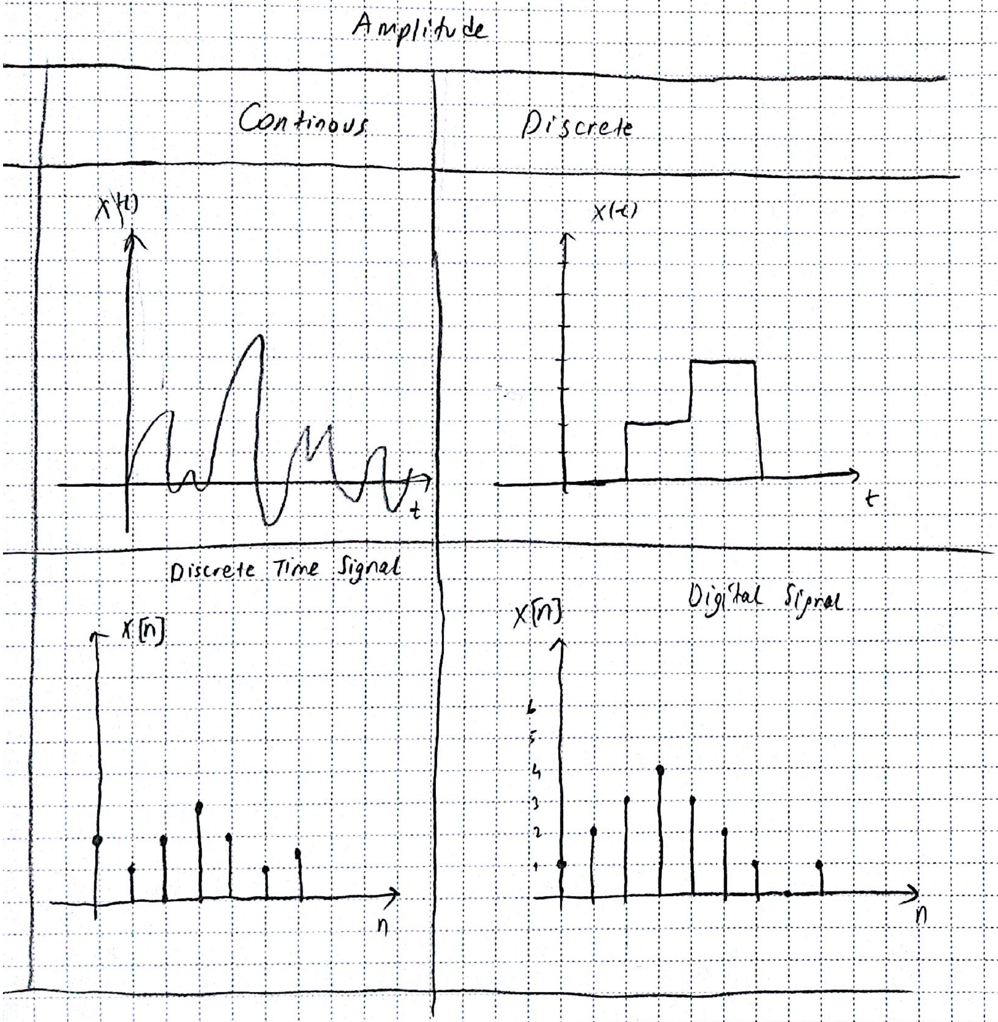
Advantages of Digital Comm. vs Analog comm.

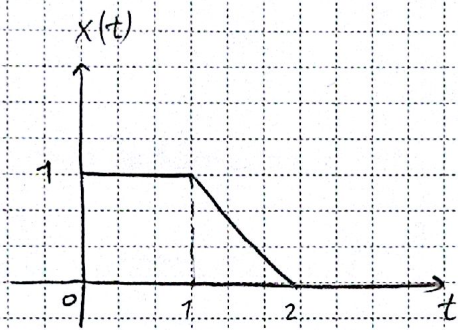
- It has more immunity →bağışıklık to noise and external interferences.
- It provides us added security to our information signal
- The error connection and detection techniques can be implemented easily.

Signals



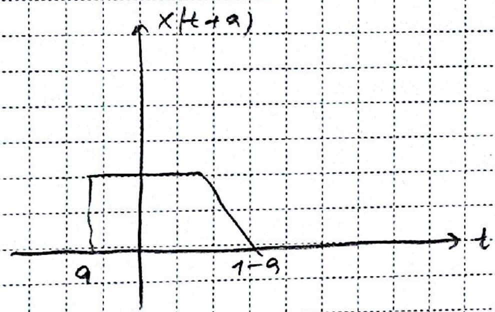
Books: - Haykin, Moher "Introduction to Analog and Digital Communications" Wiley

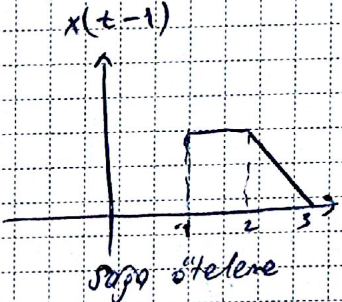
Signal Operations



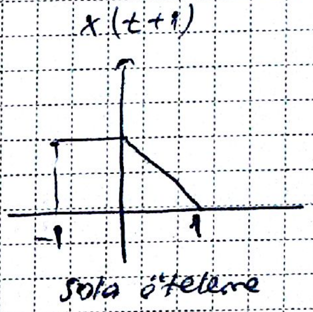
Time - Shifting (Zamanda öteleme)

a>0 x(t+a) (sola öteledi) a<0 sağa öteleme olur.



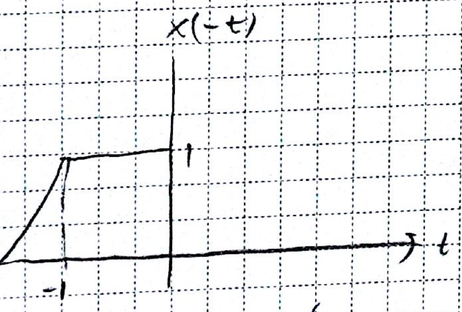


sağa öteleme

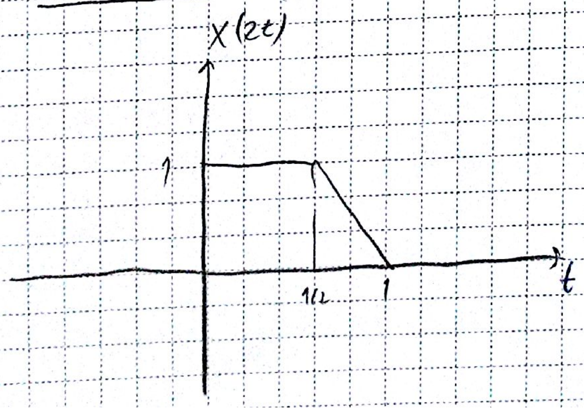


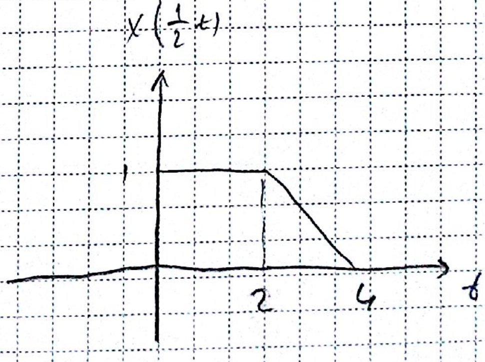
sola öteleme

Time inverse (Zamanda Tersine Alma)



Time Scaling (Zamanda ölçeklendirme)

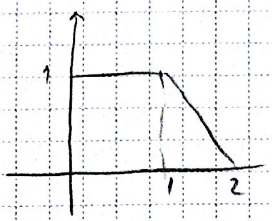




Öncelik sırası → K a S e T
scale
Tersini alın
kaydırma

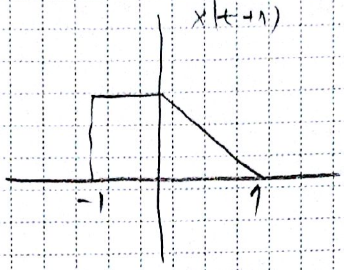
örnek = x(t)

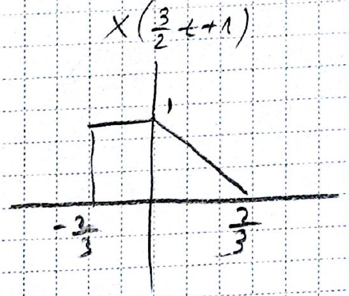
$x\left(\frac{3t}{2}+1\right) = ?$



1. yol

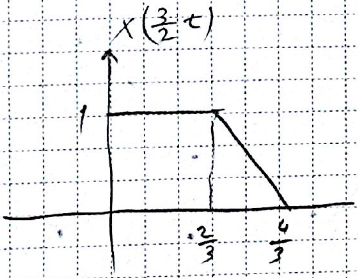
First Shifting



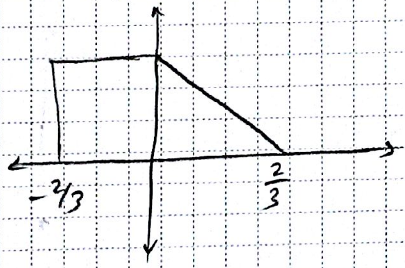


2. yol

First Scaling



$x\left(\frac{3}{2}\left(t+\frac{2}{3}\right)\right)$

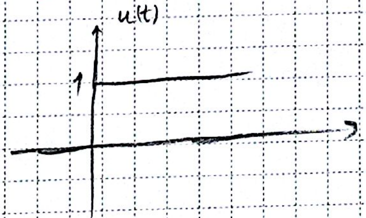


Continous Time

Some Special Signal

1- Unit Step

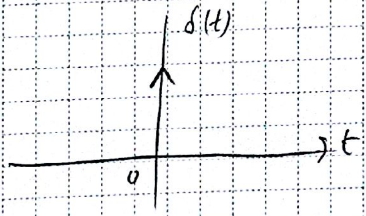
$$u(t) = \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases}$$



2- Unit impulse (Dirac delta Function)

$$\boxed{\delta(t) = \frac{du(t)}{dt}}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



Ⓧ $$\int_{-\infty}^{t} \delta(\tau).\,d\tau = \begin{cases} 1, & t\geq 0 \\ 0 & t<0 \end{cases} = u(t)$$

Ⓧ $\delta(t) = \frac{du(t)}{dt}$

$$\circledast \; f(t) \circledast \delta(t) = \int_{-\infty}^{\infty} f(\tau)\cdot\delta(\tau - t)\, d\tau = f(t)$$

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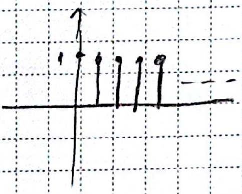
$$\circledast \; f(t)\cdot\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} x(t)\;\delta(t-a)\;dt = x(a) \qquad \text{If} \quad a=0 \qquad x(0)$$

Discrete Time

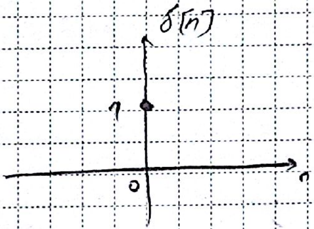
1- Unit Step $u[n] = \begin{cases} 0, & n<0 \\ 1, & n\geq 0 \end{cases}$



2- unit impulse (Kronokler Delta)

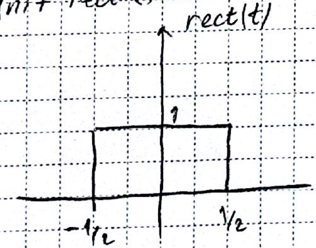
$\delta[n] = u[n] - u[n-1]$

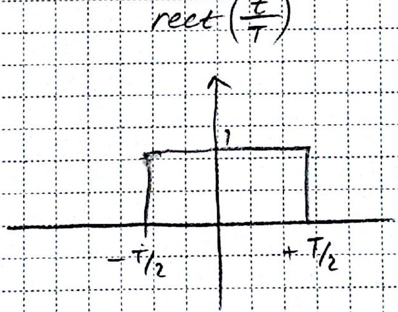
$\delta[n] = \begin{cases} 0, & n\neq 0 \\ 1, & n=0 \end{cases}$



Rectungular Pulse ($rect(t) = \Pi(t)$)

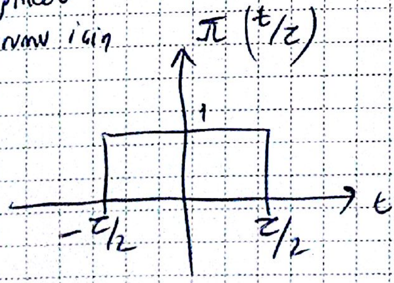
unit rect (t)





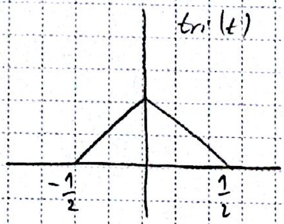
$$\Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1, & -\frac{\tau}{2} \leq t \leq \frac{\tau}{2} \\ 0, & \text{diğer} \end{cases}$$

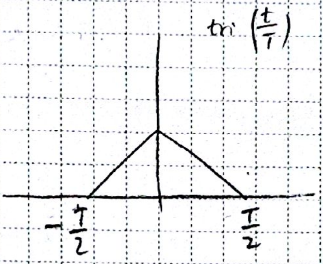
τ genişliğindeki darbe durumu için



Tringular Pulse

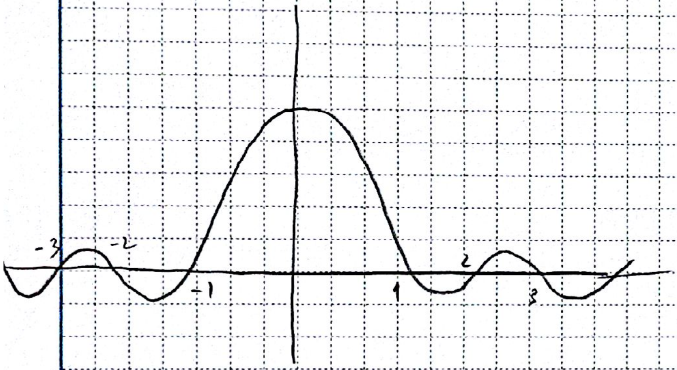
unit





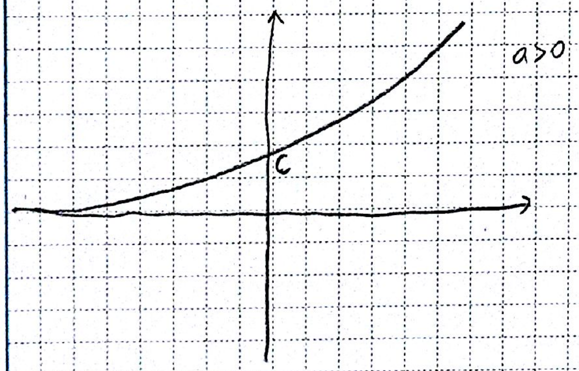
Sinc Function

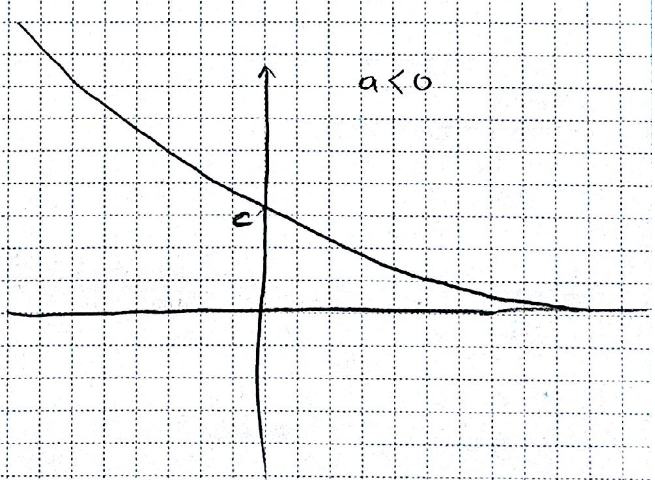
$$sinc(t) = \frac{\sin \pi t}{\pi t}$$



Exponential Signals

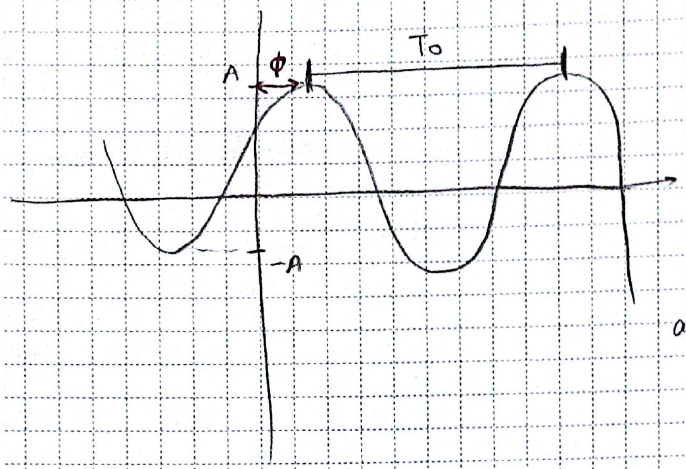
$$X(t) = C . e^{at}$$





C ve a karmasık sayıda olabilir.

Sinusoidal signals



Genlik

$X(t) = A\cos(w_0 t + \phi)$

phase

$w_0 = 2\pi f_0$

açısal frekans (rad/sn) angular freq.

Hz (frekans)

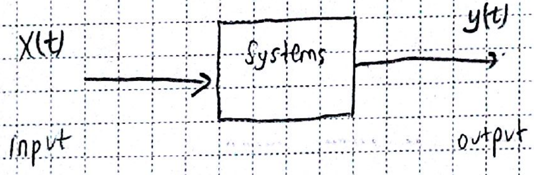
$f_0 = \frac{1}{T_0}$

Complex Exponential (Karmaşık üsteller)

euler formulas

- $e^{\pm J w_0 t} = \cos(w_0 t) \pm J \sin(w_0 t)$
- $\cos(w_0 t + \phi) = \dfrac{e^{J(w_0 t + \phi)} + e^{-J(w_0 t + \phi)}}{2}$
- $\sin(w_0 t + \phi) = \dfrac{e^{J(w_0 t + \phi)} - e^{-J(w_0 t + \phi)}}{2J}$

Systems



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Linear Time inveriant (LTI) Systems

1- Time inverieantce (zamanla değişmeyen)

$x(t) \longrightarrow y(t)$ $x(t-t_0) \longrightarrow y(t-t_0)$ time inverient denir.

— Girişteki ötelenme çıkıştakine eşit

Ex= $y(t) = \sin(x(t))$

$x_1(t) = x(t-t_0) \longrightarrow y_1(t) = \sin(x_1(t)) = \sin(x(t-t_0))$

$y_1(t) = y(t-t_0)$ bunu bularız eşitse zamanda değişmez bir sistem olur.

$y(t-t_0) = \sin x(t-t_0) = y_1(t)$ time inveriant

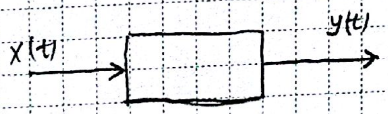
Ex = $y(t) = x(2t)$

$x_1(t) = x(t-t_0) \longrightarrow y_1(t) = x_1(2t) = x(2t-t_0)$

$y_1(t) \neq y(t-t_0) = x(2(t-t_0))$

zamanda değişe. Time verant

2- Linearity



Two proporties define a linear system.

$a \cdot x(t) \longrightarrow a \cdot y(t)$

$x_1(t) + x_2(t) \longrightarrow y_1(t) + y_2(t)$

$a\, x_1(t) + b\, x_2(t) \longrightarrow a \cdot y_1(t) + b \cdot y_2(t)$

Ex= $y(t) = t \cdot x(t)$

$$y(t) = t\left[a \cdot x_1(t) + b \cdot x_2(t)\right] = a \cdot t\, x_1(t) + b \cdot t \cdot x_2(t)$$

Unit Impulse Response of LTI system

$$x(t) = \delta(t) \longrightarrow \boxed{LTI} \longrightarrow y(t) = h(t)$$

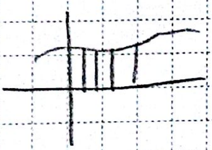
birim dürtü yanıtı
(birim dürtüye verdiği cevap)

$$\delta(t-t_0) \longrightarrow h(t-t_0)$$

$$a \cdot \delta(t-t_0) \longrightarrow a \cdot h(t-t_0)$$

$$a\,\delta(t-t_1) + b \cdot \delta(t-t_2) \longrightarrow a\, h(t-t_1) + b \cdot h(t-t_2)$$

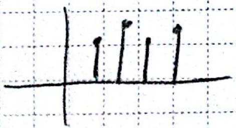
$$x(t) = \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau) \cdot d\tau \longrightarrow \underset{LTI}{\boxed{h(t)}} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$



$y(t) = x(t) \circledast h(t)$ convolution integral

$$x[n] = \sum_{n=-\infty}^{\infty} x[k] \cdot \delta[n-k] \longrightarrow \underset{LTI}{\boxed{h[n]}} \longrightarrow y[n] = \sum_{n=-\infty}^{\infty} x[k]\, h[n-k]$$

convolution sum



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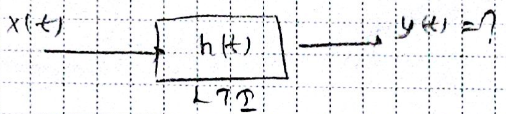
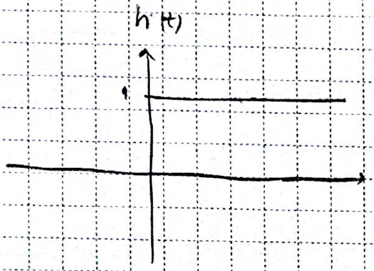
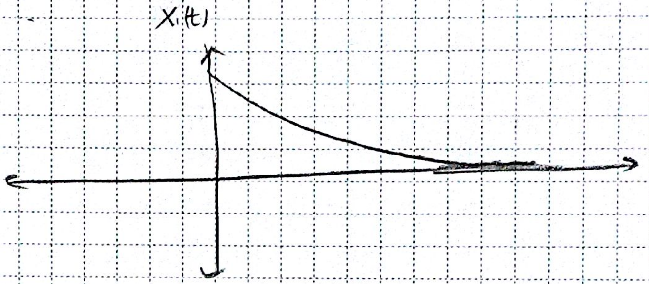
Ex = $h(t) = u(t)$ $x(t) = e^{-at} \cdot u(t)$ $a>0$

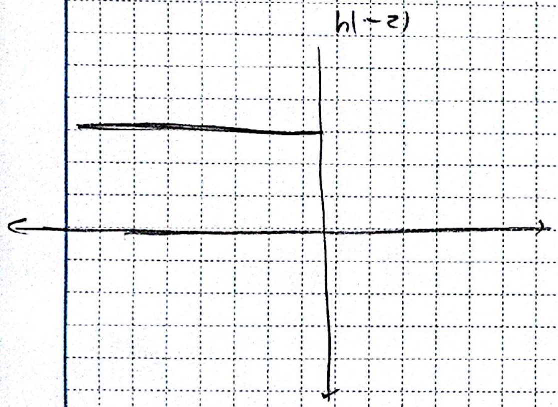


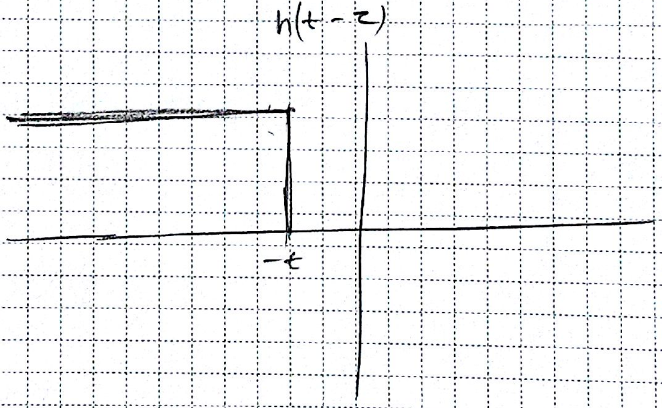
$y(t) = x(t) \circledast h(t)$





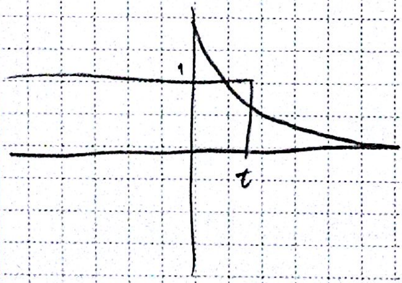
$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$$





1- $t<0$ $y(t) = 0$ örtüşme olmadığı için

2- $t>0$

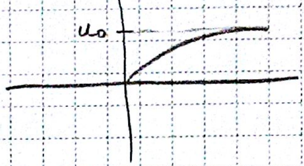


$$y(t) = \int_{0}^{t} x(\tau) \cdot h(t-\tau) \cdot d\tau = \int_{0}^{t} e^{-a\tau} \cdot 1 \cdot d\tau$$

$$= -\frac{e^{-a\tau}}{a} \Big|_{0}^{t}$$

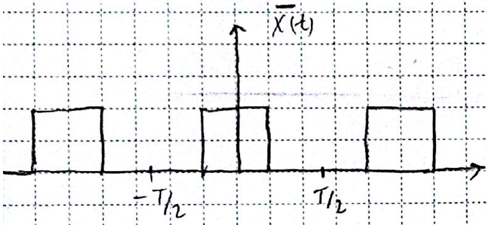
$$= \frac{1}{a}\left(1 - e^{-at}\right)$$

$$y(t) = \frac{1}{a}\left(1 - e^{-at}\right) \cdot u(t)$$

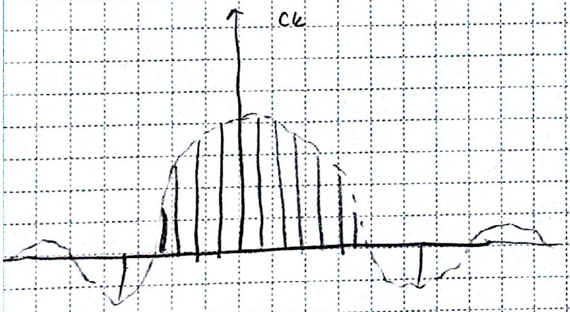


Fourier Analysis

Fourier Series

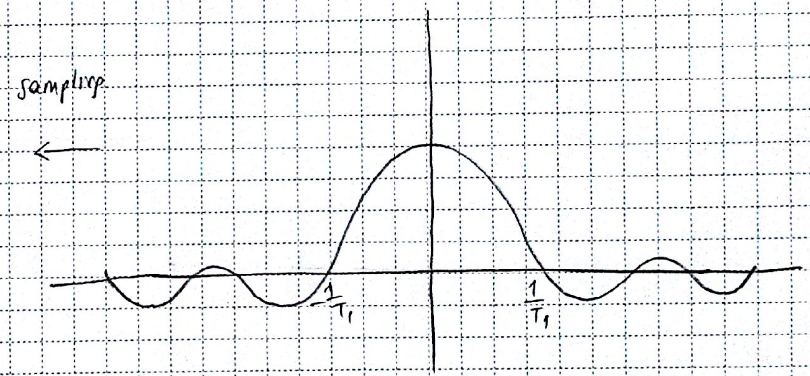


- Continous time periodic signals



Fourier Transform

- Continous time aperiodic signals



Fourier Transform (In terms of angular freq.)

$$X(t) \text{ —— } X(J\Omega)$$

sürekli zamanda genelde bu kullanılıyor.

$$X(J\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-J\Omega t}\, dt = F\{x(t)\}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(J\Omega)\, e^{J\Omega t}\, d\Omega = F^{-1}\{X(J\Omega)\}$$

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In terms of freq. f

$\Omega = 2\pi f$

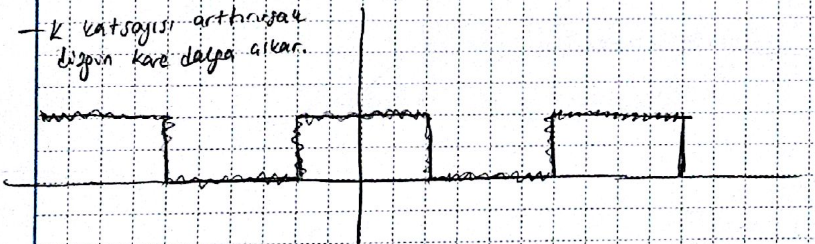
- $X(f) = \int_{-\infty}^{\infty} x(t) \, . e^{-j2\pi f t} \, dt$

- $x(t) = \int_{-\infty}^{\infty} X(f) \, e^{j2\pi f t} \, .df$

Fourier Series Representation

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} a_k \, . e^{jkw_0 t} = \sum_{k=-\infty}^{\infty} a_k \, . e^{j2\pi k . \frac{1}{T_0}} \qquad w_0 = \frac{2\pi}{T_0}$$

- K katsayısı arttırırsak düzgün kare dalga çıkar.

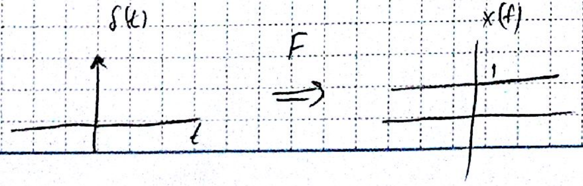


- $a_k = \frac{1}{T} \int_{-T_0/2}^{T_0/2} \tilde{x}(t) \, . e^{-j \frac{2\pi k}{T_0} . t} \, . \, dt$

Fourier Transform of unit impulse function

- $x(t) = \delta(t) \xrightarrow{\quad} X(f) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j2\pi f t} \, dt$

$$= e^{-j2\pi f t} \Big|_{t=0} = 1$$

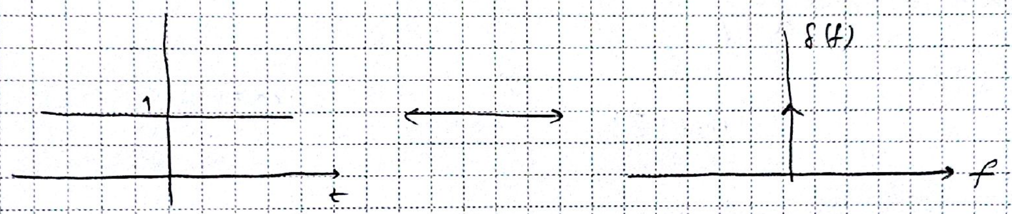


$$- \quad x(t) = \delta(t-t_0) \longleftrightarrow X(f) = \int_{-\infty}^{\infty} \delta(t-t_0)\, e^{-J2\pi f t}\, dt$$

$$= e^{-J2\pi f t_0}$$

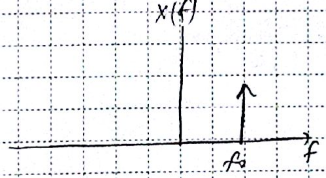
Inverse Fourier Transform of $\delta(f)$

$$F^{-1}\{\delta(f)\} = \int_{-\infty}^{\infty} \delta(f)\, e^{J2\pi f t}\, df = e^{J2\pi f t}\Big|_{f=0} = 1$$



Fourier Transform of Periodic Signals

$$X(f) = \delta(f-f_0)$$

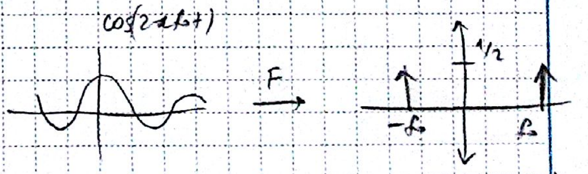


$$\boxed{e^{J2\pi f_0 t} \xleftrightarrow{F} \delta(f-f_0)}$$

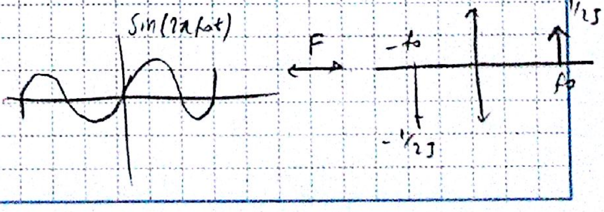
$$x(t) = F^{-1}\{X(f)\} = \int_{-\infty}^{\infty} \delta(f-f_0) \cdot e^{J2\pi f t}\, df = e^{J2\pi f_0 t}$$

$$- \quad \cos(2\pi f_0 t) = \frac{e^{J2\pi f_0} + e^{-J2\pi f_0}}{2}$$

$$*** \quad \cos(2\pi f_0 t) \xleftrightarrow{F} \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right] \Rightarrow$$



$$** \quad \sin(2\pi f_0 t) \xleftrightarrow{F} \frac{1}{2J}\left[\delta(f-f_0) - \delta(f+f_0)\right] \Rightarrow$$



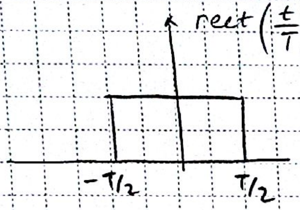
In General

$$X(t) = \sum_{k=-\infty}^{\infty} a_k \, e^{j2\pi f_0 t k} \longleftrightarrow X(f) = \sum_{k=-\infty}^{\infty} a_k \cdot F\left\{ e^{jk2\pi f_0 t} \right\}$$

$$= \sum_{k=-\infty}^{\infty} a_k \, \delta(f - kf_0)$$

17/02/2016

Fourier Transform of rect $\left(\frac{t}{T}\right)$



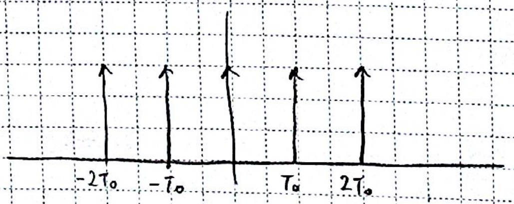
$$F\left\{ rect\left(\frac{t}{T}\right)\right\} = \int_{-\infty}^{\infty} rect\left(\frac{t}{T}\right) e^{-j2\pi f t} \cdot dt$$

$$= \int_{-T/2}^{T/2} e^{-j2\pi f t} \cdot dt = -\frac{1}{j2\pi f} \, e^{-j2\pi f t} \Big|_{-T/2}^{T/2} = \frac{1}{j2\pi f}\left[e^{j2\pi f \, T/2} - e^{-j2\pi f T/} \right.$$

$$= \frac{1}{\pi f} \sin(\pi f T) = T \, \frac{\sin(\pi f T)}{\pi f T} = T \cdot sinc(T f)$$

Fourier Transform of impulse train

$$x(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT_0)$$



Tarih: 17/02/2016

Fourier Series representation of impulse train

$$X(t) = \sum_{k=-\infty}^{\infty} a_k \cdot e^{J2\pi k f_0 t}$$

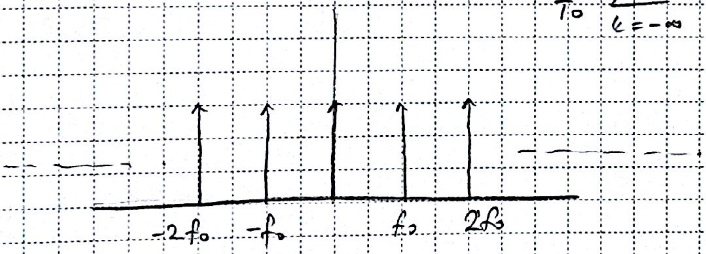
$$a_k = \frac{1}{T_0}\int_{-T/2}^{T/2} x(t)\; e^{-J2\pi f_0 t}\, dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)\; e^{-J2\pi k f_0 t}\, dt$$

$$= \frac{1}{T_0}\; e^{-2\pi k f_0 t}\Big|_{t=0} = \frac{1}{T_0}$$

$$X(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T_0}\, e^{J2\pi f_0 t}$$

$$F\{x(t)\} = \frac{1}{T_0}\, F\left\{\sum_{k=-\infty}^{\infty} e^{J2\pi f_0 t}\right\} = \frac{1}{T_0}\sum_{k=-\infty}^{\infty} F\left\{e^{J2\pi k f_0 t}\right\}$$

$$= \frac{1}{T_0}\sum_{k=-\infty}^{\infty} \delta(f - k f_0) \qquad f_0 = \frac{1}{T_0}$$



$$\boxed{X(t) = \sum_{m=-\infty}^{\infty} \delta(t - m T_0) \xleftrightarrow{F} \frac{1}{T_0}\sum_{k=-\infty}^{\infty} \delta(f - k f_0)}$$

$\propto$

Properties of Fourier Transform

1- Linearity

$$a \cdot x_1(t) + b\, x_2(t) \xleftrightarrow{F} a\, X_1(f) + b\, X_2(f)$$

2- Time Scaling

$$x(t) \xleftrightarrow{F} X(f)$$

$$x(a \cdot t) \xleftrightarrow{F} F\{x(at)\} = \int_{-\infty}^{\infty} x(at)\, e^{-j2\pi ft}\, dt$$

$z = at$

$dz = a\,dt$

$$= \int_{-\infty}^{\infty} x(z) \cdot e^{-j\frac{2\pi f z}{a}} \frac{dz}{|a|}$$

$$= \frac{1}{|a|} X\left(\frac{f}{a}\right)$$

$$\boxed{x(at) \xleftrightarrow{F} \frac{1}{|a|} X \frac{f}{a}}$$

3- Duality

Tarih: 17 / 02 /2016

4- Time Shifting

$$g(t) \xleftrightarrow{F} G(f)$$

$$F\{g(t-t_0)\} = \int_{-\infty}^{\infty} g(t-t_0)\cdot e^{-J2\pi ft}\, dt \qquad \begin{matrix} \tau = t-t_0 \\ d\tau = dt \end{matrix}$$

$$= \int_{-\infty}^{\infty} g(\tau)\; e^{-J2\pi f(\tau+t_0)}\, d\tau$$

$$= e^{-J2\pi f t_0} \underbrace{\int_{-\infty}^{\infty} g(\tau)\cdot e^{-J2\pi f\tau}\, d\tau}_{G(f)}$$

$$\boxed{g(t-t_0) \xleftrightarrow{F} e^{-J2\pi f t_0}\cdot G(f)}$$

5- Frequency Shifting

$$g(t) \xleftrightarrow{F} G(f)$$

$$e^{J2\pi f_0 t}\cdot g(t) \xleftrightarrow{F} G(f-f_0)$$

$$F\left\{e^{-J2\pi f_0 t}\; g(t)\right\} = \int_{-\infty}^{\infty} g(t)\cdot e^{-J2\pi(f+f_0)t}\, dt = G(f+f_0)$$

Ex=

$$g(t) = \mathrm{rect}\left(\frac{t}{T}\right)\; \cos(2\pi f_c t) \qquad F\{g(t)\} = ? \qquad G(f) = ?$$

$$G(f) = F\{g(t)\} = F\left\{\mathrm{rect}\left(\frac{t}{T}\right)\cdot \cos 2\pi f_c t\right\}$$

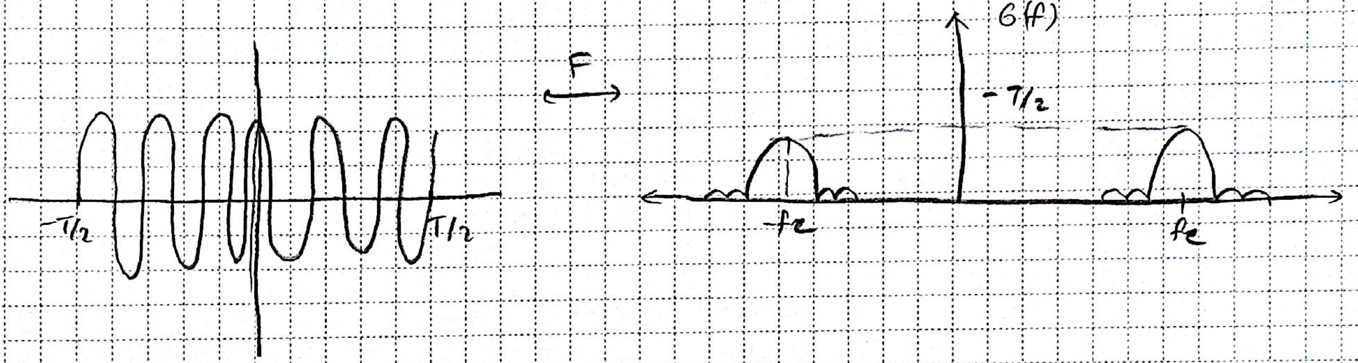
$$= F\left\{\mathrm{rect}\left(\frac{t}{T}\right)\cdot \frac{1}{2}\left(e^{J2\pi f_c t} + e^{-J2\pi f_c t}\right)\right\}$$

$$= \frac{1}{2}\, F\left\{\mathrm{rect}\left(\frac{t}{T}\right) e^{J2\pi f_c t} + \mathrm{rect}\left(\frac{t}{T}\right) e^{-J2\pi f_c t}\right\}$$

$$\text{rect}\left(\frac{t}{T}\right) \xleftrightarrow{F} T\ \text{sinc}\left(f\,T\right)$$

$$G(f) = \frac{1}{2}\ F\left\{ \text{rect}\left(\frac{t}{T}\right) e^{J2\pi f_c t} \right\} + \frac{1}{2}\ F\left\{ \text{rect}\left(\frac{t}{T}\right) e^{-J2\pi f_c t} \right\}$$

$$G(f) = \frac{1}{2} \cdot T \cdot \text{sinc}\left(T(f-f_c) \right) + \frac{1}{2}\ T\ \text{sinc}\ (f+f_c)$$



6- Differentiation in time domain

$g(t) \xleftrightarrow{F} G(f)$

$t \longrightarrow \pm\infty \qquad g(t) \longrightarrow 0$

$$\frac{d\,g(t)}{dt} \xleftrightarrow{F} J2\pi f\ G(f)$$

$$\frac{d^n g(t)}{dt^n} \xleftrightarrow{F} (J2\pi f)^n \cdot G(f)$$

7- Modulation Theorem

$g_1(t) \longleftrightarrow G_1(f) \qquad g_2(t) \longleftrightarrow G_2(f)$

$$g_1(t) \cdot g_2(t) \xleftrightarrow{F} G_1(f) \circledast G_2(f) = \int_{-\infty}^{\infty} G_1(\lambda) \cdot G_2(f-\lambda)\ d\lambda$$

α

$$g_1(t) \cdot g_2(t) \longleftrightarrow G_1(f) \circledast G_2(f)$$

Proof.

$$F\left\{ g_1(t) \cdot g_2(t) \right\} = \int_{-\infty}^{\infty} g_1(t) \cdot g_2(t) \cdot e^{-J2\pi f t} \, dt$$

$$g_2(t) = \int_{-\infty}^{\infty} G_2(f) \cdot e^{+J2\pi f t} \, df$$

$$= \int_{-\infty}^{\infty} g_1(t) \int_{-\infty}^{\infty} G_2(f) \; e^{-J2\pi (f_0 - f') t} \, dt \, df'$$

$\lambda = f - f'$

$d\lambda = -df'$

$$= \int_{-\infty}^{\infty} G_2(f-\lambda) \int_{-\infty}^{\infty} g_1(t) \; e^{-J2\pi \lambda t} \, dt \, d\lambda$$

$$= \int_{-\infty}^{\infty} G_2(f-\lambda) \; G_1(\lambda) \, d\lambda = G_1(f) \circledast G_2(f)$$

Example: $\text{rect}\left(\frac{t}{T}\right) \cos(2\pi f_c t)$ finds fft, use modulation theorem.

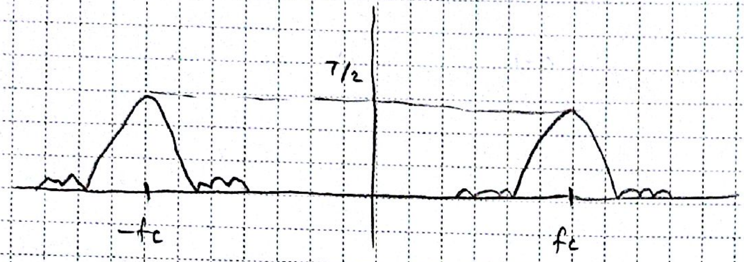
$$= F\left\{ \text{rect}\left(\frac{t}{T}\right) \right\} \circledast F\left\{ \cos(2\pi f_c t) \right\}$$

$$= T \cdot \text{sinc}(fT) \circledast \frac{1}{2} \left(\delta(f-f_c) + \delta(f+f_c) \right)$$

$$= \frac{T}{2} \cdot \text{sinc}(fT) \circledast \delta(f-f_c) + \frac{T}{2} \text{sinc}(fT) \circledast \delta(f+f_c)$$

*** $f(x) \circledast \delta(x-x_0) = \int_{-\infty}^{\infty} f(x-\tau) \cdot \delta(\tau - x_0) \cdot d\tau = f(x-\tau) \Big|_{\tau = x_0} = f(x - x_0)$

$$= \frac{1}{2}\left[T\cdot sinc((f-f_c)T) + T\ sinc((f+f_c)T) \right]$$



8- Convolution ~~Theorem~~ Theorem

$$g_1(t) \longleftrightarrow G_1(f) \qquad g_2(t) \longleftrightarrow G_2(f)$$

$$g_1(t) \circledast g_2(t) \overset{F}{\longleftrightarrow} G_1(f)\cdot G_2(f)$$

Proof.

$$F\left\{g_1(t)\circledast g_2(t)\right\} = F\left\{\int_{-\infty}^{\infty} g_1(\tau)\cdot g_2(t-\tau)\ d\tau\right\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_1(\tau)\cdot g_2(t-\tau)\cdot e^{-j2\pi ft}\, d\tau . d$$

$$= \int_{-\infty}^{\infty} g_1(\tau)\cdot \underbrace{\int_{-\infty}^{\infty} g_2(t-\tau)\ e^{-j2\pi ft} .dt}\ .d\tau = \int_{-\infty}^{\infty} g_1(\tau)\cdot G_2(f)\cdot e^{-j2\pi f\tau} .d\tau$$

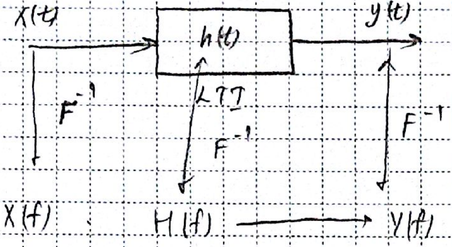
$$= G_2(f)\int_{-\infty}^{\infty} g_1(\tau)\ e^{-j2\pi f\tau} .d\tau$$

$$= G_1(f) . G_2(f)$$

$\propto$

Example: $h(t) = e^{-at} \cdot u(t)$, $x(t) = e^{-bt} \cdot u(t)$ $a,b > 0$

Find the response of the system to the input signal $x(t)$ using convolution theorem.



$$X(f) = \frac{1}{b+j2\pi f} \qquad H(f) = \frac{1}{a+j2\pi f}$$

$$Y(f) = X(f) \cdot H(f) = \frac{1}{(b+j2\pi f)} \cdot \frac{1}{(a+j2\pi f)}$$

$$= \frac{A}{a+j2\pi f} + \frac{B}{b+j2\pi f}$$

$$A = \frac{1}{(a+j2\pi f)(b+j2\pi f)} (a+j2\pi f) \Bigg|_{f=-\frac{a}{j2\pi}} = \frac{1}{b - j2\pi \frac{a}{j2\pi}} = \frac{1}{b-a}$$

$$B = \frac{1}{(a+j2\pi f)(b+j2\pi f)} (b+j2\pi f) \Bigg|_{f=-\frac{b}{j2\pi}} = -\frac{1}{b-a}$$

$$Y(f) = \frac{\frac{1}{b-a}}{a+j2\pi f} - \frac{\frac{1}{b-a}}{b+j2\pi f}$$

$$= \frac{1}{b-a} \left[\frac{1}{a+j2\pi f} - \frac{1}{b+j2\pi f} \right]$$

$$F^{-1}\{Y(f)\} = y(t)$$

↘

$$F^{-1}\{Y(f)\} = \frac{1}{b-a}\left[F^{-1}\left\{\frac{1}{a+J2\pi f}\right\} - f^{-1}\left\{\frac{1}{b+J2\pi f}\right\}\right]$$

$$\boxed{y(t) = \frac{1}{b-a}\left[e^{-at}.u(t) - e^{-bt}.u(t)\right]}$$

9-) Parseval Theorem

$$g(t) \longleftrightarrow G(f)$$

$$\int_{-\infty}^{\infty} |g(t)|^2 \, dt = \int_{-\infty}^{\infty} |G(f)|^2 \cdot df$$

Energy is parseval after transformation.

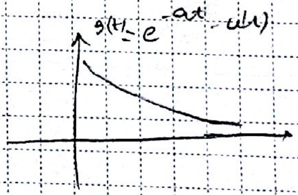
$$|g(t)|^2 = g(t) \cdot g^*(t)$$

Energy Signal

$$E_g = \int_{-\infty}^{\infty} g(t) \cdot g^*(t) \, dt = \int_{-\infty}^{\infty} |g(t)|^2 \, dt < \infty$$

Example =

$g(t) = e^{-at} \cdot u(t)$ $a > 0$ Energy of $g(t) = ?$



$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 \cdot dt = \int_{0}^{\infty} e^{-2at} \cdot dt$$

$$= \frac{1}{2a}$$

Tarih: 23 / 02 /2016

Eneryg of freq. domain

Enery of $G(f) = \int_{-\infty}^{\infty} |G(f)|^2 . df$

$g(t) \xrightarrow{F} G(f) = \frac{1}{a+j2\pi f}$ $\qquad G^*(f) = \frac{1}{a-j2\pi f}$

$$E_y = \int_{-\infty}^{\infty} |G(f)|^2 . df = \int_{-\infty}^{\infty} G(f) . G^*(f) . df$$

$$= \int_{-\infty}^{\infty} \frac{1}{a+j2\pi f} . \frac{1}{a-j2\pi f} df = \int_{-\infty}^{\infty} \frac{1}{a^2+(2\pi f)^2} df$$

$$= \frac{1}{2\pi a} \arctan\left(\frac{2\pi}{a} f\right)\Big|_{-\infty}^{\infty} = \boxed{\frac{1}{2a}}$$

Autocorrelation function

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t) \, x^*(t-\tau) \, dt$$

Energy of the signal $E_y = R_x(0) = \int_{-\infty}^{\infty} x(t) \, x^*(t) \, dt$

Energy spectral density (ESD) $S_x(f) = \int_{-\infty}^{\infty} R_x(\tau) . e^{-j2\pi f\tau} d\tau$

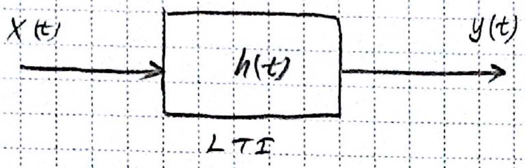
$$R_x(\tau) = \int_{-\infty}^{\infty} S_x(f) . e^{j2\pi f\tau} df$$

$E_y = R_x(0) = \int_{-\infty}^{\infty} S_x(f) . df$ (altında kalan alan hesaplayarak)

3 tane enerji hesaplama yöntemi var.

$\rightarrow \boxed{S_x(f) = |X(f)|^2}$

Effect Filtering on ESD



$$Y(f) = X(f) \cdot H(f)$$

$$|Y(f)|^2 = |X(f)|^2 \cdot |H(f)|^2$$

$$S_y(f) = S_x(f) \cdot |H(f)|^2$$

Cross Correlation of Energy Signal

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) \cdot y^{*}(t-\tau) \cdot dt$$

$$R_{yx}(\tau) = \int_{-\infty}^{\infty} y(t)\, x^{*}(t-\tau)\, dt$$

$$R_{xy}(\tau) = R_{xy}^{*}(-\tau)$$

Birbirlerine doğrudan eşit değiller.

If $R_{xy}(0) = \int_{-\infty}^{\infty} x(t) \cdot y^{*}(t)\, dt = 0$ $x(t)$ and $y(t)$ are said to be orthogonal.

Cross Spectral Density $S_{xy}(f) = \int_{-\infty}^{\infty} R_{xy}(\tau) \cdot e^{-j2\pi f\tau}\, d\tau$

$$S_{yx}(f) = \int_{-\infty}^{\infty} R_{yx}(\tau) \cdot e^{-j2\pi f\tau}\, d\tau$$

$\propto$

Power Spectral Density

Power Signal $$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \cdot dt \quad < \infty$$

Train:

$$x_T(t) = x(t)\cdot \text{rect}\left(\frac{t}{T}\right) = \begin{cases} x(t), & -T/2 \leq t \leq T/2 \\ 0, & \text{otherwise} \end{cases}$$

$$x_T(t) \xleftrightarrow{F} X_T(f)$$

$$\int_{-\infty}^{\infty} |x_T(t)|^2 \, dt = \int_{-\infty}^{\infty} |X_T(f)|^2 \cdot df.$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x_T(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{\infty} |X_T(f)|^2 \, df$$

If $|X_T(f)|^2$ approaches infinty at the same rate as T the integral is converged

$$P = \int_{-T/2}^{T/2} \underbrace{\lim_{T\to\infty} \frac{1}{T} |X(f)|^2}_{\downarrow} \cdot df$$

$$P = \int_{-T/2}^{+T/2} S_X(f) \cdot df$$

Power Spectral Density = $S_X(f) = \lim_{T\to\infty} \frac{1}{T} |X(f)|^2$

$\propto$

Example = $x(t) = g(t) \cdot \cos(2\pi f_c t) \qquad S_x(f) = ?$

$$\cos(2\pi f_c t) \xleftrightarrow{F} \frac{1}{2}\left(\delta(f-f_c) + \delta(f+f_c)\right)$$

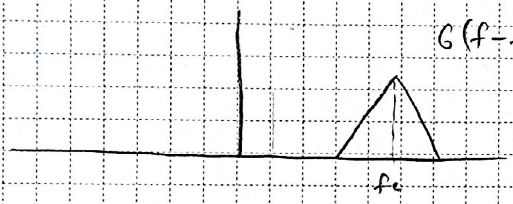
$$x(t) = g(t) \cdot \cos(2\pi f_c t) \xleftrightarrow{F} G(f) \circledast \frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

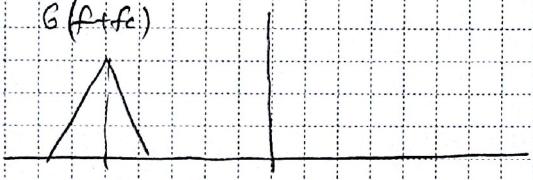
$$= \frac{1}{2}\left[G(f-f_c) + G(f+f_c)\right]$$

$$|X(f)|^2 = \frac{1}{4}\left[G(f-f_c) + G(f+f_c)\right]^2$$

$$= \frac{1}{4}\left[|G(f-f_c)|^2 + \overset{0}{\cancel{2\;G(f-f_c)\;G(f+f_c)}} + |G(f+f_c)|^2\right]$$

ikisinin çarpımı 0' olur. Grafikleri çarpışmaz.



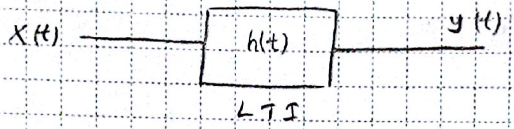


$$= \frac{1}{4}\left[|G(f-f_c)|^2 + |G(f+f_c)|^2\right]$$

$$S_x(f) = |X(f)|^2 = \frac{1}{4}\left[S_x(f-f_c) + S_x(f+f_c)\right]$$

$$S_x(f) = \lim_{T\to\infty} |X(f)|^2$$

Effect of filtering on PSD



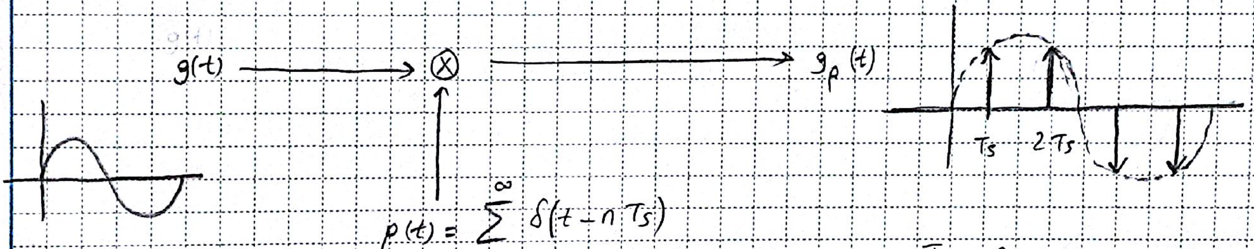
$$S_y(f) = S_x(f)\,|H(f)|^2$$

24/02/2016

Sampling Theorem

1. Ideal impulse train sampling
2. Natural Sampling

1- Ideal impulse Train Sampling



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

T_s : Sampling period

$f_s = \frac{1}{T_s}$ sampling frequency or rate

$$g_p(t) = g(t) \cdot p(t) = g(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$g_p(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s)$$

α

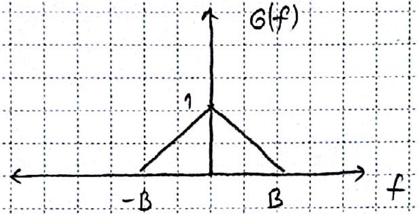
In frequency Domain

$$G_p(f) = F\{g_p(t)\} = F\{g(t) \cdot p(t)\} = G(f) \circledast P(f)$$

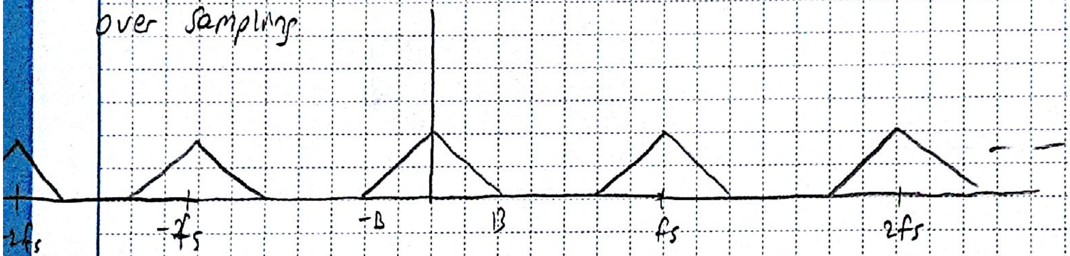
$$P(f) = f_s \cdot \sum_{k=-\infty}^{\infty} \delta(f - k f_s)$$

$$G_p(f) = G(f) \circledast f_s \sum_{k=-\infty}^{\infty} \delta(f - k f_s) = f_s \sum_{k=-\infty}^{\infty} G(f) \circledast \delta(f - k f_s)$$

$$G_p(f) = f_s \sum_{k=-\infty}^{\infty} G(f - k f_s)$$

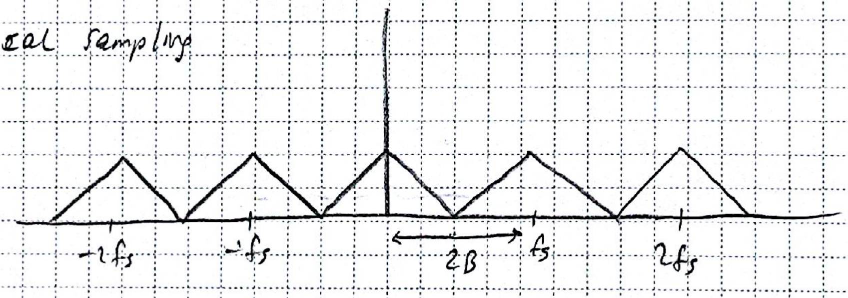


over sampling



$f_s \geq 2B$

Critical sampling

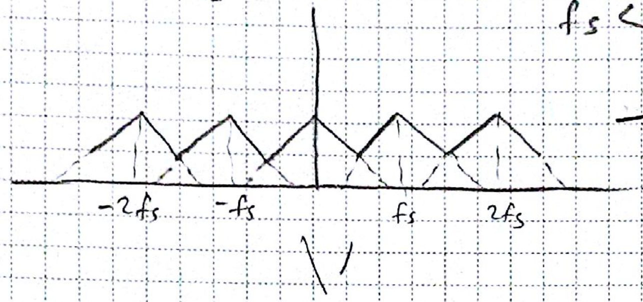


$f_s = 2B$

$\propto$

under sampling

$f_s < 2B$ → aliasing effect

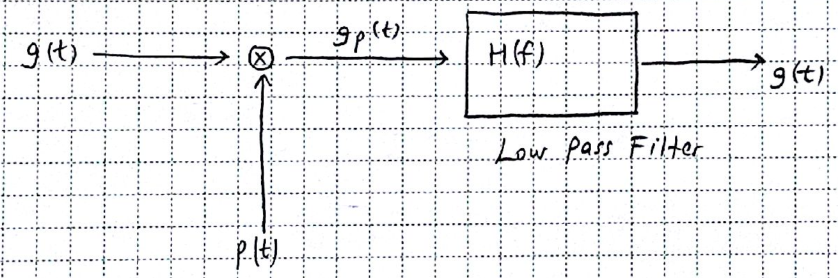


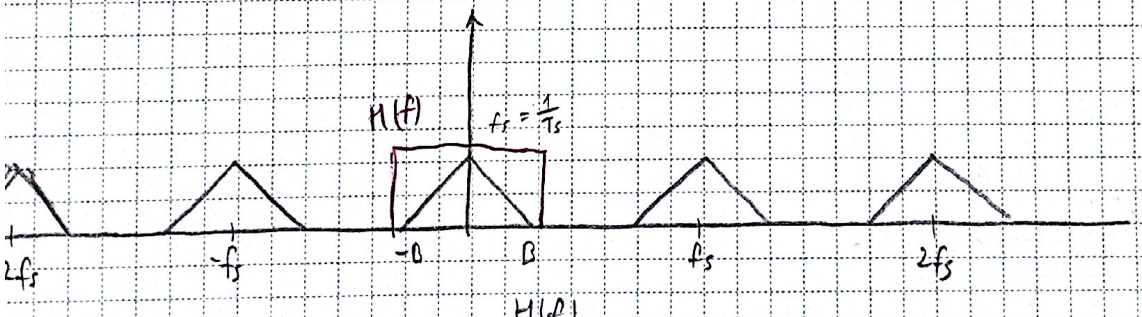
Sampling Theorem

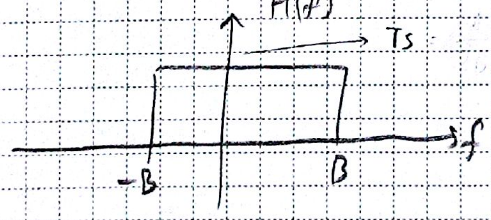
Let $g(t)$ be a band-limited signal, $G(f) = 0$ for $|f| > B$, then $g(t)$ is uniquely determined by its samples $g(nT_s)$ $n = 0, \mp 1, \mp 2, \mp 3, —$ if

$$f_s > 2B \quad \text{where} \quad f_s = \frac{1}{T_s}$$

Whittaker, Shannon, Nyquist

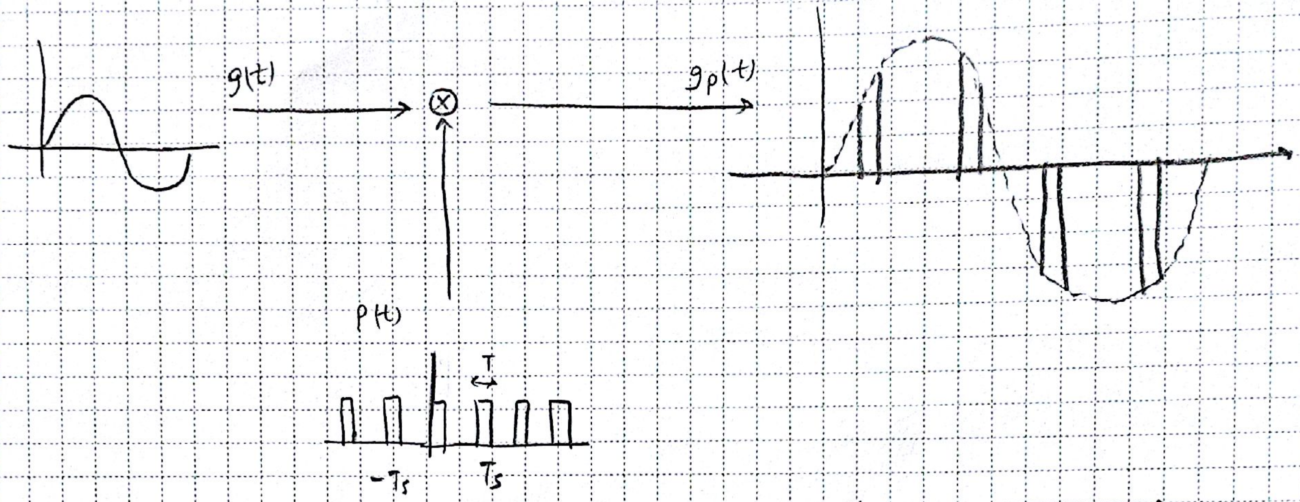






Natural Sampling

$g_p(t) = g(t) \cdot p(t)$ p(t) is periodic rectungular pulse train



Fourier Series representation of p(t) $p(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{+J2\pi n f_s t}$ where

$$C_n = \frac{1}{T_s} \text{sinc}\left(\frac{nT}{T_s}\right)$$

$$C_n = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} e^{-J2\pi n f_s t} \cdot dt$$

$$g_p(t) = g(t) \cdot p(t)$$

$$= g(t) \sum_{n=-\infty}^{\infty} C_n \cdot e^{J2\pi n f_s t}$$

$$= \sum_{n=-\infty}^{\infty} C_n \cdot g(t) \cdot e^{J2\pi n f_s t}$$

$$G_p(f) = F\left\{ g_p(t) \right\} = \sum_{n=-\infty}^{\infty} C_n \cdot F\left\{ g(t) \cdot e^{J2\pi n f_s t} \right|$$

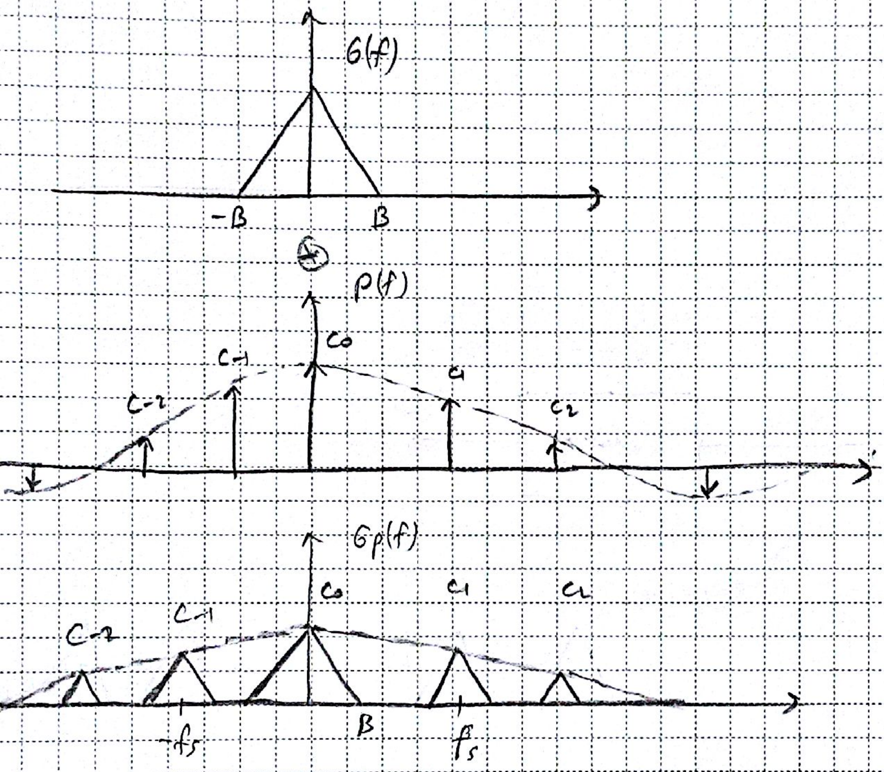
$\propto$ →

Tarih: 24/02/2016

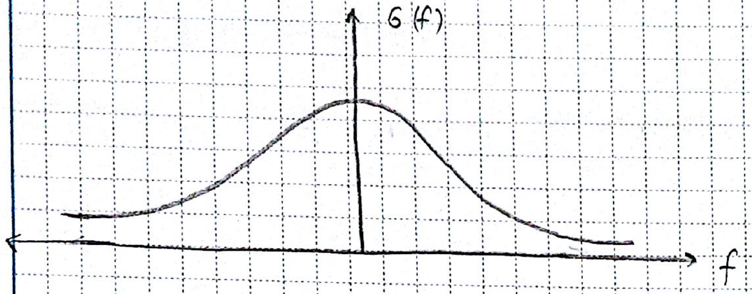
$$G_p(f) = \sum_{n=-\infty}^{\infty} c_n \, G(f - n \cdot f_s)$$

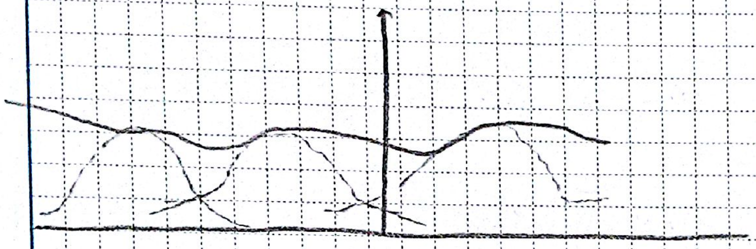
Compare to ideal sampling $\Big\rangle$ aradaki fark bir c_n katsayısı bulunmaktadır.

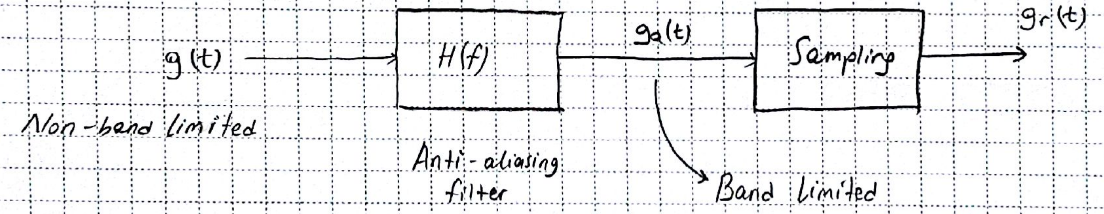
$$G_p(t) = \sum_{k=-\infty}^{\infty} G(f - k f_s)$$

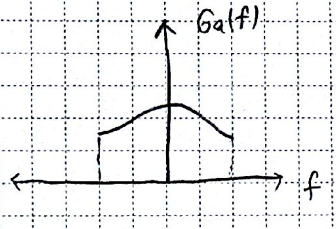


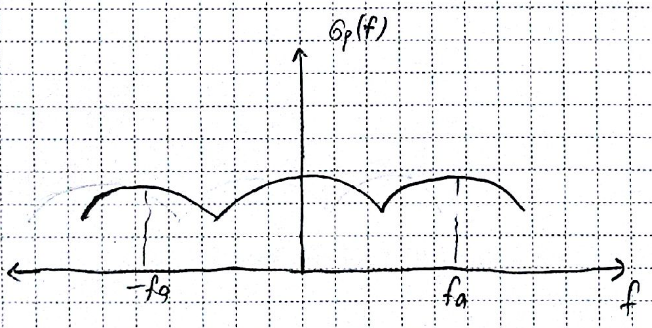
Pratical issues in Sampling Anti- aliasing











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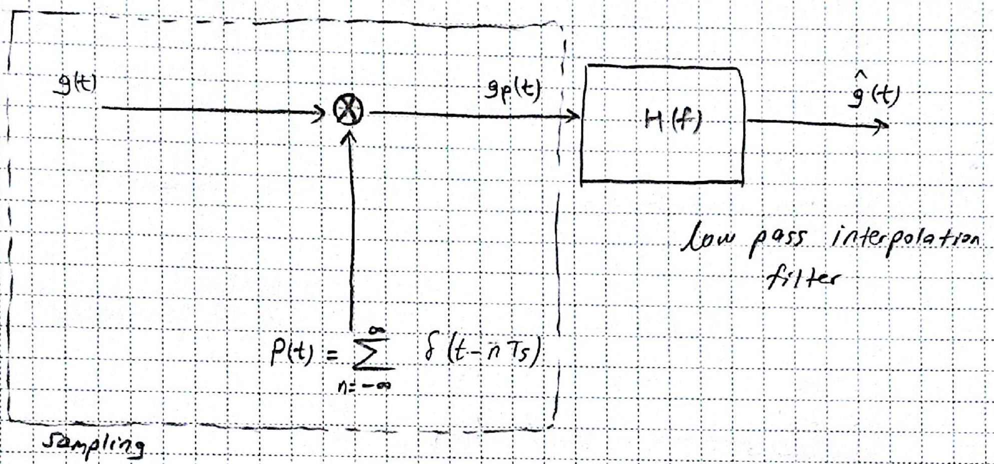
- Prior to sampling, a low pass anti-alias filter is used to attenuate those high frequency components of a message signal.

- The filtered signal is sampled at a rate slightly higher than Nyquist rate.

Reconstruction of signal from its samples



In frequency

$$G_p(f) = f_s \sum_{k=-\infty}^{\infty} G(f - kf_s)$$

to be filtered out by low pass filter

$$= f_s\, G(f) + \cancel{f_s \sum_{k\neq 0} G(f - kf_s)}$$

$$\tilde{G}(f) = H(f) \cdot G_p(f) = G(f)$$

If $G(f) = 0$, $|f| > W$ and $f_s = 2W$

$$H(f) = \begin{cases} c, & |f| < W \\ 0, & \text{otherwise} \end{cases}$$

$$c \cdot f_s\, G(f) = G(f) \Rightarrow \boxed{c = \frac{1}{f_s} = \frac{1}{2W}}$$

Tarih: 01/03/2016

In time:

$$\hat{g}(t) = g_p(t) \circledast h(t)$$

$$= \left(\sum_{n=-\infty}^{\infty} g(nT_s) \; \delta(t - nT_s) \right) \circledast h(t)$$

$$= \sum_{n=-\infty}^{\infty} g(nT_s) \left[\delta(t - nT_s) \circledast h(t) \right]$$

$$= \sum_{n=-\infty}^{\infty} g(nT_s) \cdot \underbrace{h(t - nT_s)}_{\text{interpolation filter}}$$

$h(t) = F^{-1}\{H(f)\}$

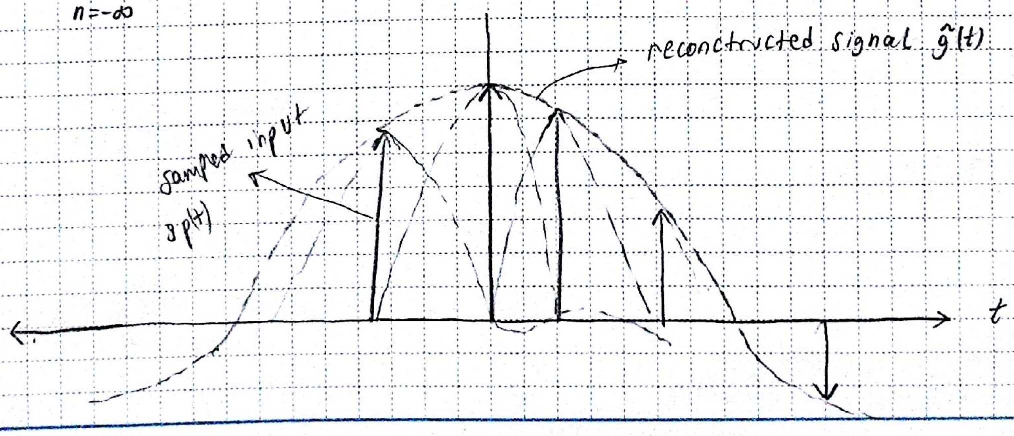
$\boxed{h(t) = \text{sinc}(2wt)}$

$\circledast$ - tarafta değerleri var
nedensel değil. filtre gerçekte
kullanılamaz.

$$\tilde{g}(t) = \sum_{n=-\infty}^{\infty} g(nT_s) \cdot \text{sinc}(2w(t - nT_s))$$

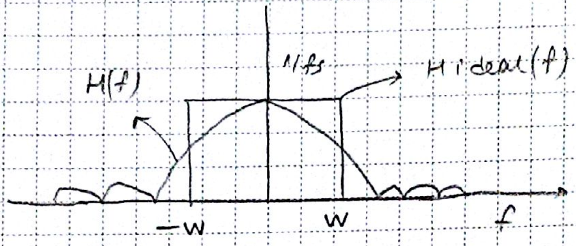
Since sampled at Nyquist rate $f_s = \frac{1}{T_s} = 2w$ $\quad T_s = \frac{1}{2w}$

$$\tilde{g}(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2w}\right) \text{sinc}(2wt - n) \rightarrow \text{interpolation formula}$$



h(t) is non-causal

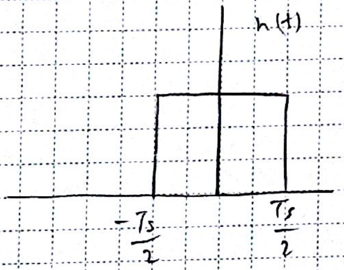
Pratial interpolation:



$$H(f) = \frac{1}{f_s} \operatorname{sinc}\left(\frac{f}{2f_s}\right)$$

At Nyquist rate $H(f) = \frac{1}{2w} \operatorname{sinc}\left(\frac{f}{4w}\right)$

In impulse response $h(t) = \operatorname{rect}\left(\frac{t}{T_s}\right) = \operatorname{rect}(2wt)$

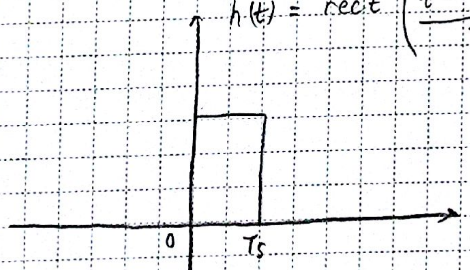


zero-order hold filter

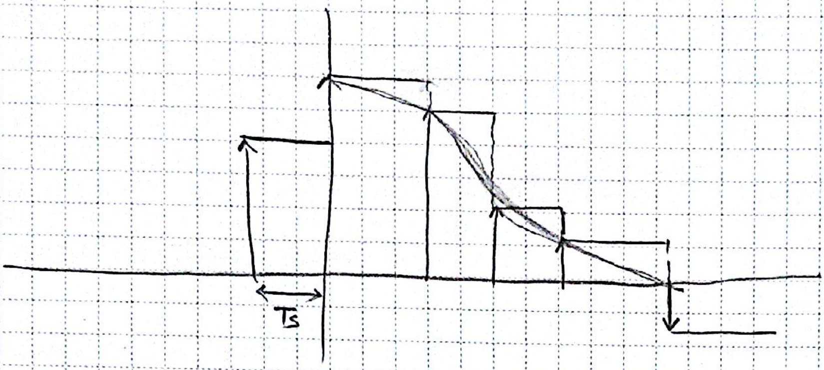
Interpolation Formula

$$\tilde{g}(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2w}\right) \operatorname{rect}(2wt-n)$$

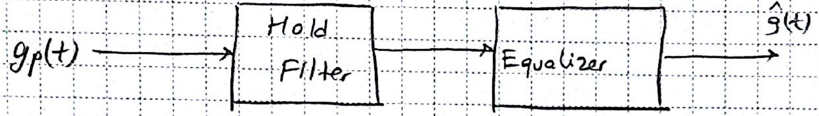
$$h(t) = \operatorname{rect}\left(\frac{t - T_s/2}{T_s}\right)$$

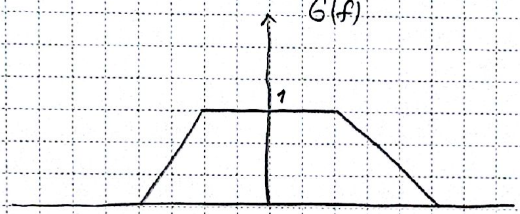


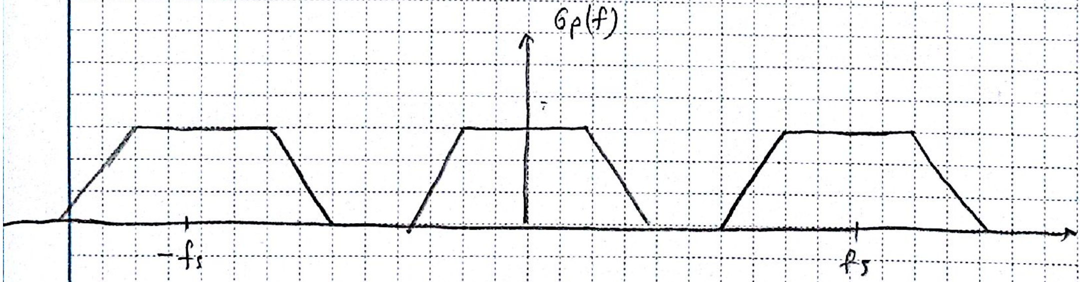
Tarih: 01 / 03 /2016

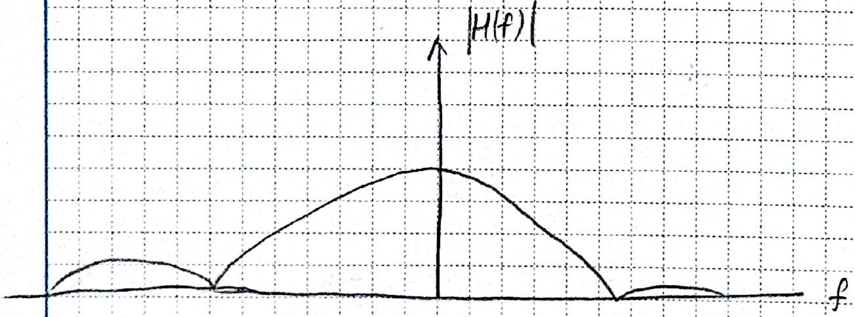


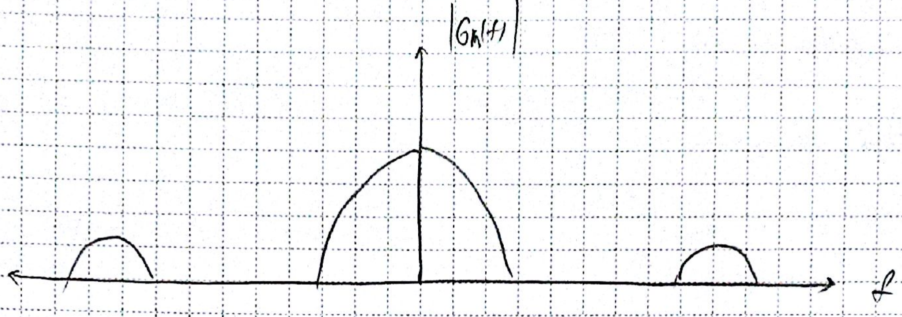
Equalization

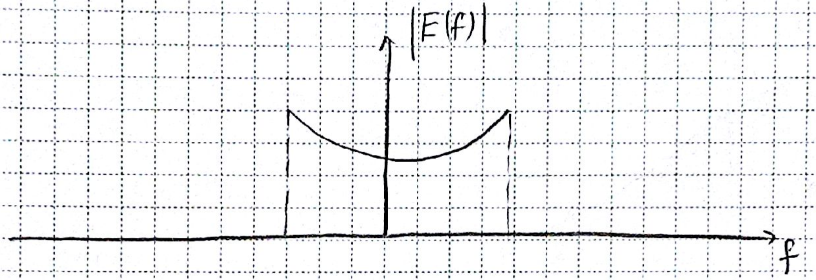




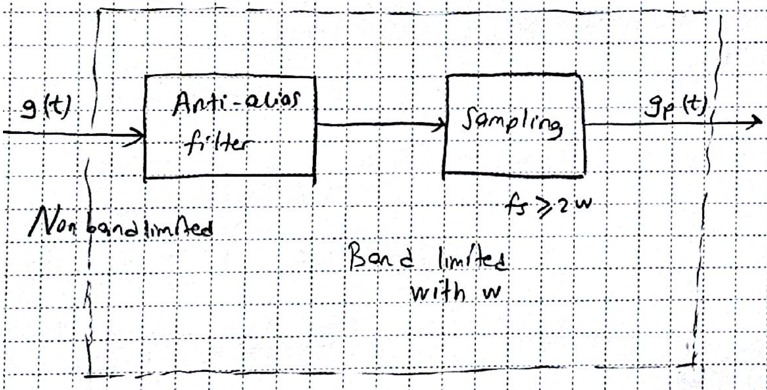




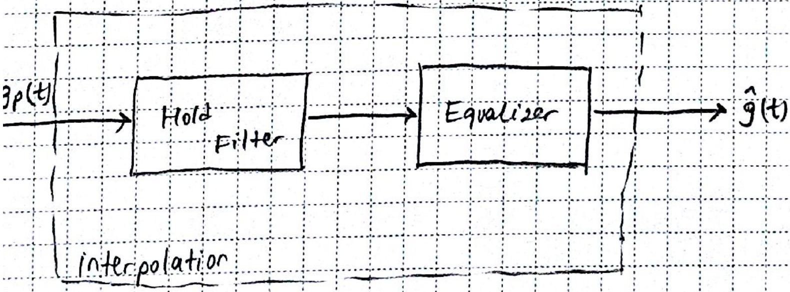




Overwiew of Pratical Sampling



Continous / Discrete Conversion

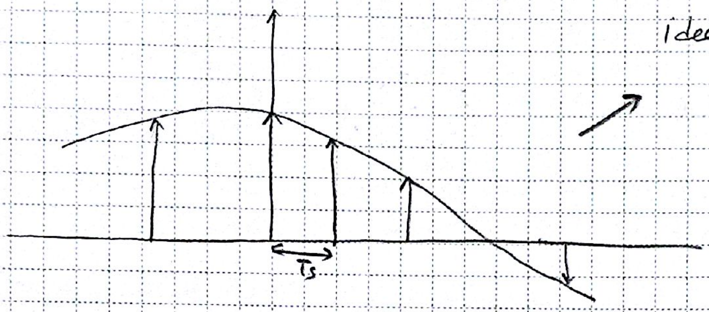


Discrete / Continous Conversion

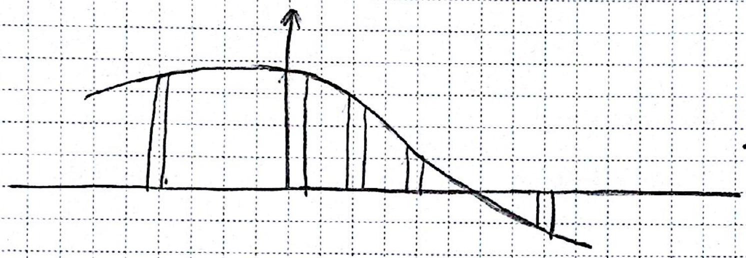
Tarih: 01 / 03 /2016

Pulse Modulation

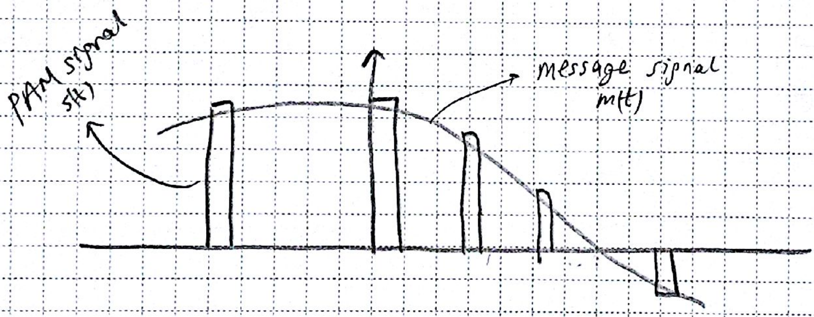
PAM (Pulse Amplitude Modulation)



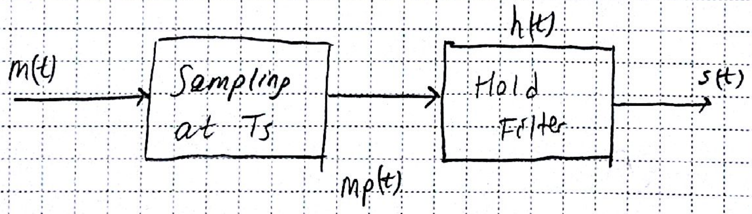
→ ideal sampling



→ Natural sampling



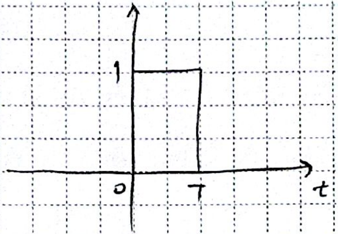
→ PAM
(or flat top sampling)



$$m_p(t) = m(t) \cdot \underbrace{p(t)}_{\text{impulse train}} = \sum_{n=-\infty}^{\infty} m(nT_s)\,\delta(t - nT_s)$$

$$s(t) = m_p(t) \circledast h(t) = \sum_{n=-\infty}^{\infty} m(nT_s) \cdot h(t - nT_s)$$

Hold filter impulse response



$$h(t) = \text{rect}\left(\frac{t - T/2}{T}\right)$$

Frequency Domain Analysis of Sample and hold filter

$$s(t) = m_p(t) \circledast h(t)$$

$$S(f) = M_p(f) \cdot H(f)$$

$$M_p(f) = F\{m_p(t)\} = F\{m(t) \cdot p(t)\} = F\{m(t)\} \; F\{p(t)\}$$

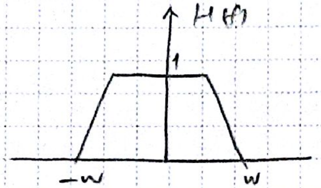
$$= M(f) \circledast P(f)$$

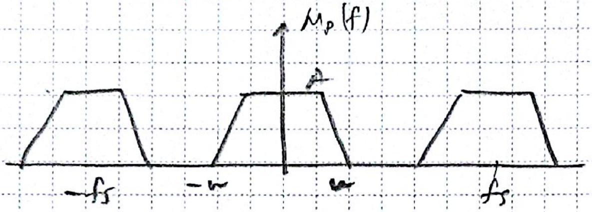
$$= M(f) \circledast f_s \sum_{k=-\infty}^{\infty} \delta(f - kf_s) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)$$

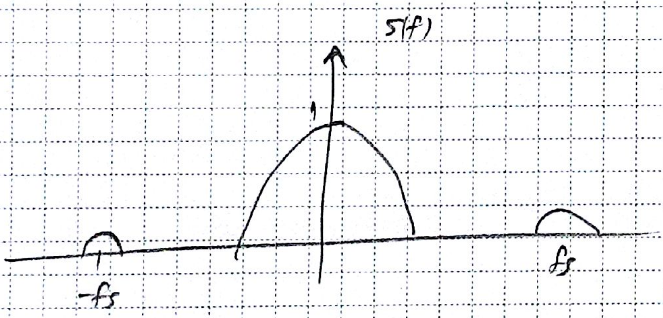
$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \cdot H(f)$$

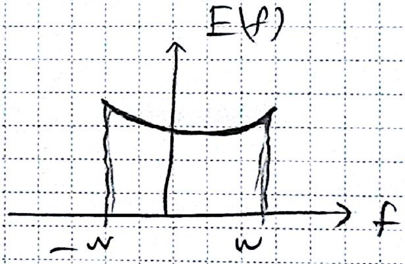
$$H(f) = F\left\{\text{rect}\left(\frac{t - T/2}{T}\right)\right\} = T_s \; \text{sinc}(f T_s) \; e^{-j2\pi T_s}$$

$$= \frac{1}{f_s} \; \text{sinc}\left(\frac{f}{f_s}\right) e^{-j\frac{2\pi}{f_s}}$$

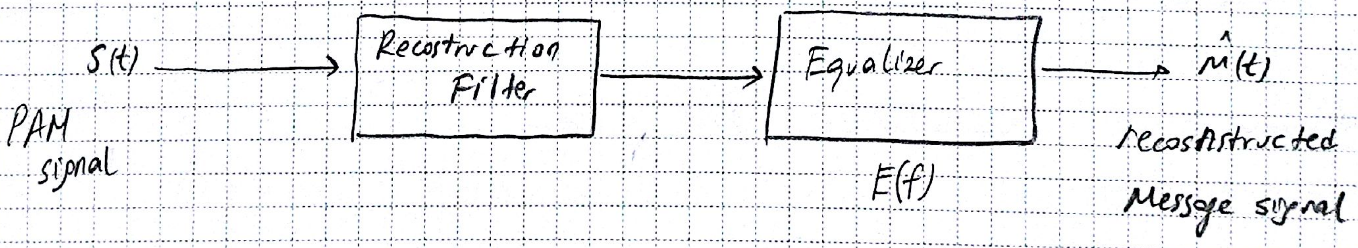








Reconstruction of PAM Signal



If the amplitude response of hold-filter is

$$H(f) = |T_s \cdot sinc(f \; T_s)|$$

them the amplitude response of equalizer should be

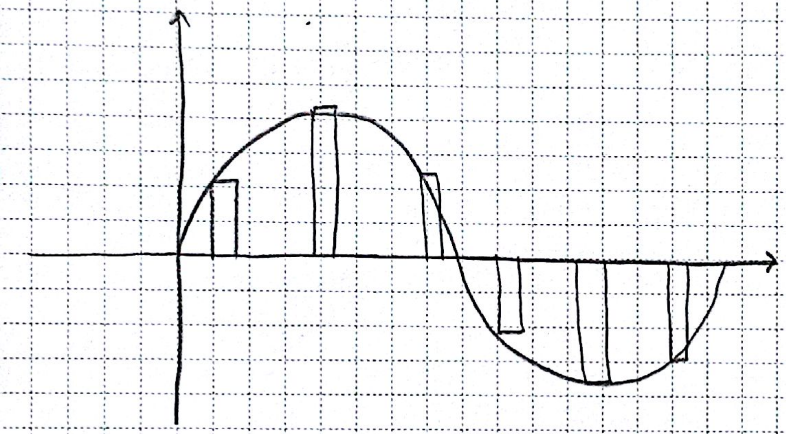
$$|E(f)| = \frac{1}{|H(f)|} = \frac{1}{|T_s \operatorname{sinc}(f T_s)|}$$

Pulse Modulation Techniques

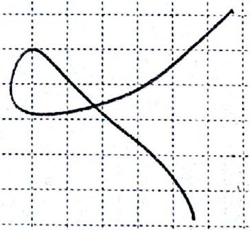
1- PAM

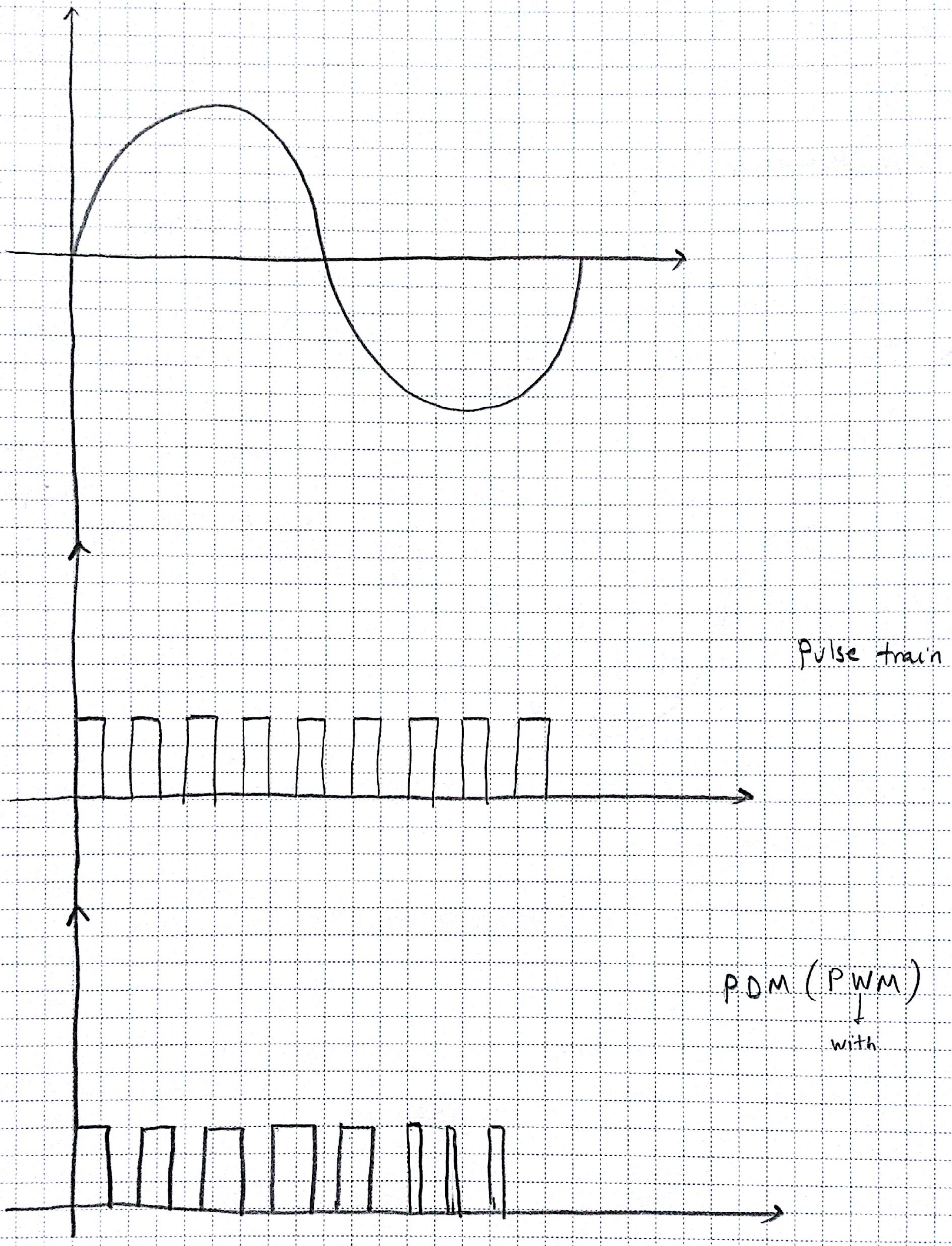
2- PDM : Pulse Duration Mod.

3- PPM : Pulse position Mod.

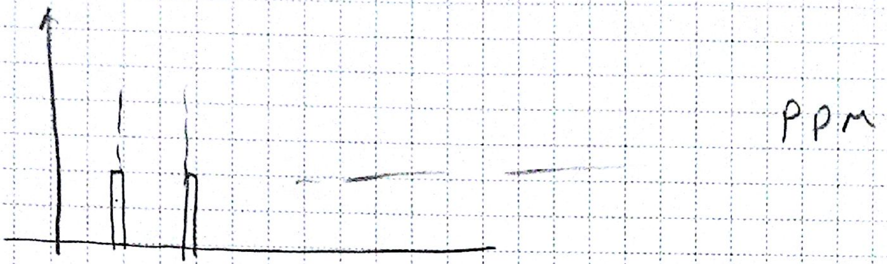


P AM





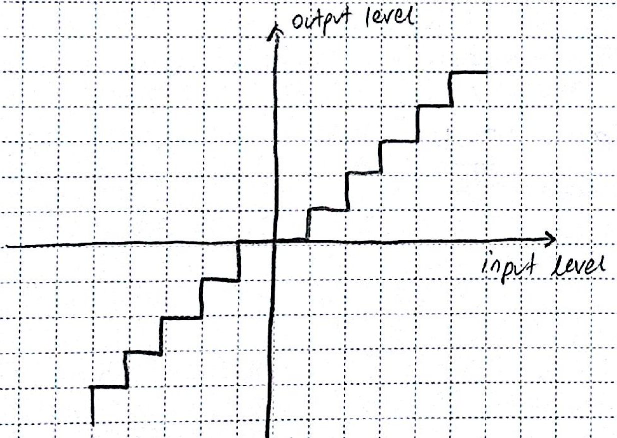
PDM: The samples of the message signal are used to vary the duration of the individual pulses.



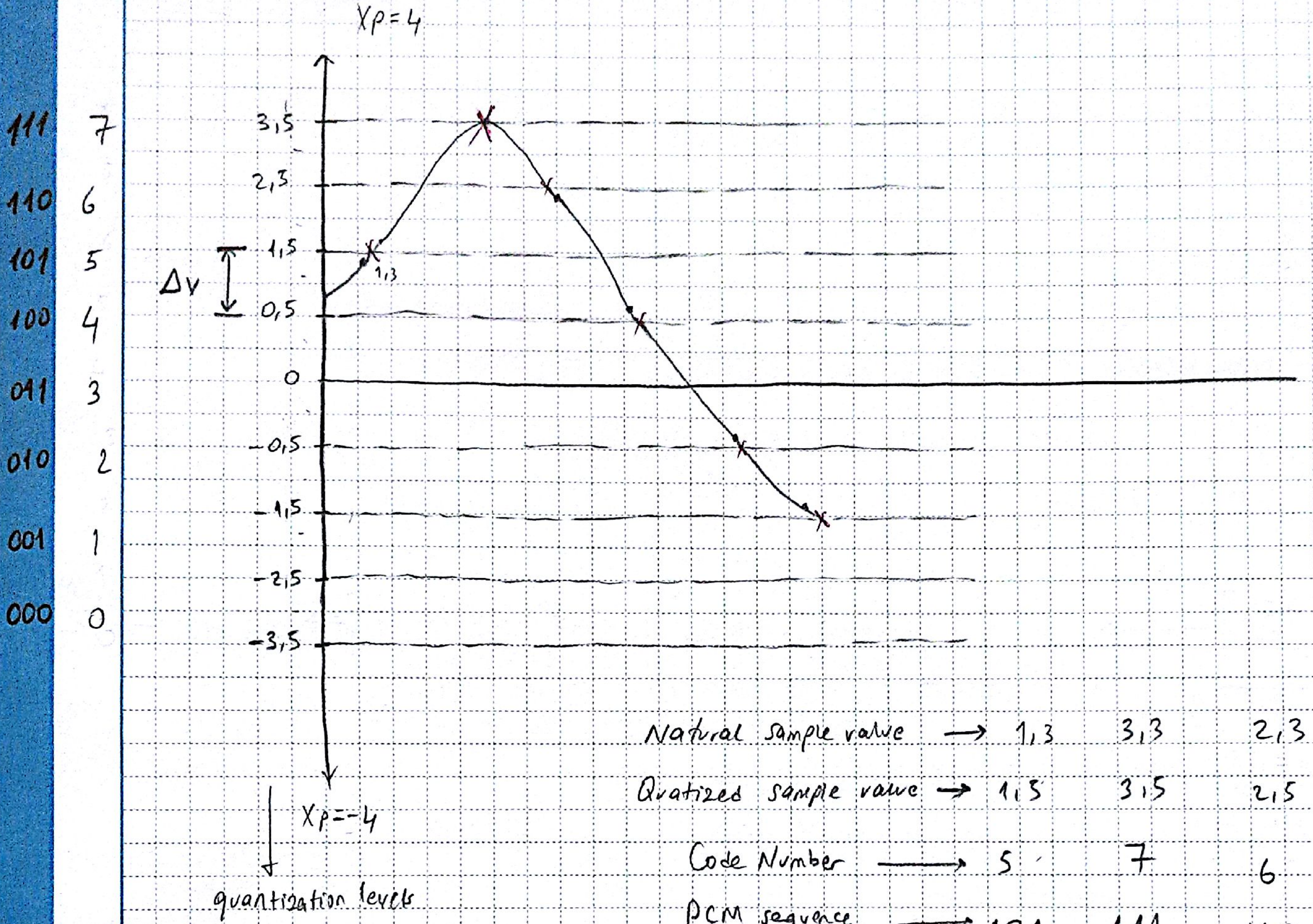
Pulse Code Modulation

A message signal is represented by a squence of coded pulses

Quantization = The process of transforming the sample amplitude of a baseband signal into a discrete amplitude taken from a finite set of possible levels.



Tarih: 02 / 03 /2016



Natural sample value →	1,3	3,3	2,3
Quatized sample value →	1,5	3,5	2,5
Code Number →	5	7	6
PCM sequence →	101	111	110

Quantization step size $= \Delta v = \frac{2 x_p}{L} = \frac{2 \cdot 4}{8} = 1$

L : Number of Levels

$L = 2^m = 2^3 = 8$

Number of bits : $m = \log_2 L$

Example: Telephone

Human voice bandwith $B = 60 - 7000$ Hz

Teleplione " " $B = 3400 +$ quand band $= 4$kHz

Sampling nate (at Nyquist) $f_s = 2B = 8$kHz

$L = 256$

$m = \log_2 L = \log_2 256 = 8$ bits/sample

Tarih: 02 / 03 /2016

Bit rate $BR = m \cdot f_s = 8 \cdot 8000 = 64\,000$ bps $= 64$ kbps

08/03/2016

Example = Compact ~~d~~ ~~~~elise

$B = 20\,kHz$

Nyquist sampling rate : $f_N = 2B = 40\,kHz$

Sampling $f_s = 1.0025$ $f_N = 44,1\,kHz$

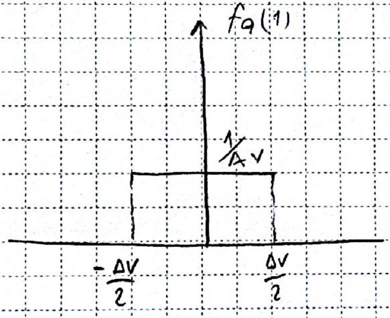
Quantization level = $L = 65\,536 = 2^{16}$

Number of bit/sample = $m = \log_2 L = 16$ bit/sample

Bit rate= $BR = m \cdot f_s = 16 \cdot (44,100) = 1,411$ Mbps

Quantizatron Error

Uniform quantization



$$\Delta V = \frac{2 \times p}{L}$$

Maximum Error = $\frac{\Delta V}{2}$

Quantization Error = $q \sim u\left[-\frac{\Delta V}{2}, \frac{\Delta V}{2}\right]$

Mean of Quantization error = $E[q] = \int_{-\infty}^{\infty} q\, f_N(q)\, dq = \int_{-\Delta V/2}^{\Delta V/2} q \frac{1}{\Delta V} \cdot dq$

$$= \frac{q^2}{2\Delta V}\Bigg|_{-\frac{\Delta V}{2}}^{\Delta V/2} = \frac{1}{2\Delta V}\left(\left(\frac{\Delta V}{2}\right)^2 - \left(\frac{\Delta V}{2}\right)\right) = 0$$

$$Var(q) = \sigma_q^2 = E\left[(q - E[q])^2\right] = E[q^2]$$

$$= \int_{-\frac{\Delta V}{2}}^{\frac{\Delta V}{2}} q^2 \cdot \frac{1}{\Delta V} \cdot dq = \frac{1}{3\Delta V} q^3 \Bigg|_{-\frac{\Delta V}{2}}^{\frac{\Delta V}{2}} = \frac{1}{3\Delta V}\left(\left(\frac{\Delta V}{2}\right)^3 + \left(\frac{\Delta V}{2}\right)^3\right)$$

$$\sigma_q^2 = \frac{(\Delta V)^2}{12}$$

$$SNR \triangleq \frac{\text{Signal power}}{\text{Noise power}} = \frac{P_x}{\sigma_A^2}$$

PA: power of signal x(t)

$$\Delta V = \frac{2X_p}{L} \longrightarrow \sigma_q^2 = \frac{X_p^2}{3L^2}$$

$$L = 2^m \qquad \sigma_a^2 = \frac{X_p^2}{3.2^{2m}}$$

$$\boxed{3\times 2^{2m}}$$

SNR in [dB]

$$SNR = 10\log_{10}\frac{P_x}{\sigma_a^2} = 10\log_{10}\frac{P_x}{X_p^2}\times 3\times 2^m$$

$$= 10\log\left(\frac{3P_x}{X_p^2}\right) + 20\,m\log_{10}2$$

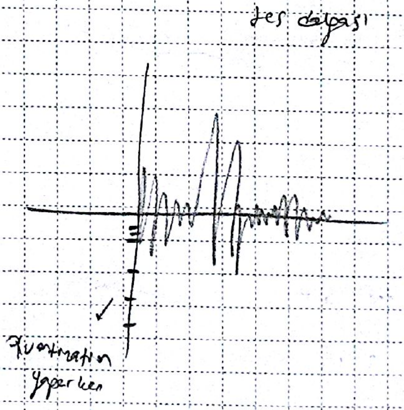
$$SNR = C + 6.02\,m \quad dB$$

1 bit is increased, 6.02 dB gain in SNR

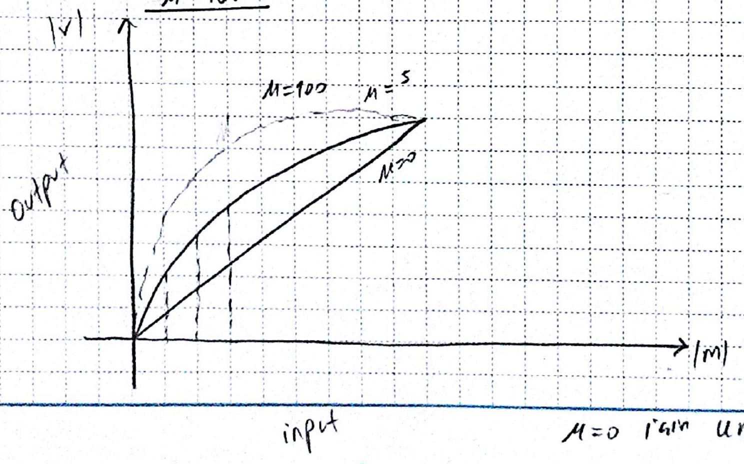
Non-uniform Quantization

μ-low (North America and Japan)

A-low (European and rest of the world)



μ-low



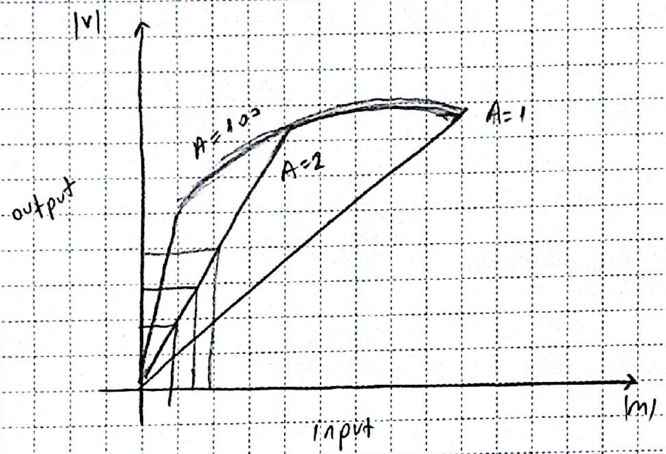
μ=0 için uniform olur

$$|v| = \frac{\log(1+\mu|m|)}{\log(1+\mu)}$$

* $\mu=0$ uniform quantization

$$|v| = \frac{Vt}{V_{max}}$$

A - law



$$|v| = \begin{cases} \frac{A|m|}{1+\log(A)}, & 0 \leq m \leq \frac{1}{A} \\ \frac{1+\log(A|m|)}{1+\log(A)}, & \frac{1}{A} \leq |m| \leq 1 \end{cases}$$

Transmission Bandwith in PCM

Quantization level $L=2^m$

Number of bits/sample = $m=\log_2 L$

Sampling rate = $fs \geqslant 2B$

Bandwith of the signal : B

Transmission bandwith = $B_T = \frac{m fs}{2}$ [Hz]

If sampled at Nyquist frequency : $B_T = \frac{m 2B}{2} = mB$

Tarih: 08/03/2016

Example = A message signal m(t) is bandlimited to 4 kHz sampled at a rate that is 15 times higer than Nyquist rate 256 level uniform quantization is used what is the transmission bandwith of P(t) signal

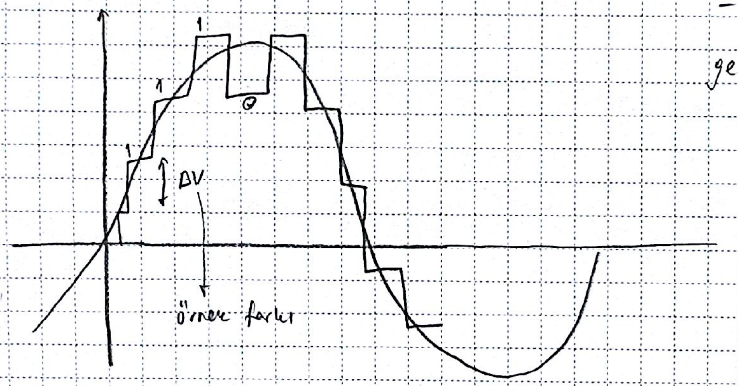
Nyquist rate $f_N = 2B = 2 \times 4000 = 8kHz$

Sampling rate $f_s = 1.5 \times f_N = 1,5 \times 8000 = 12kHz$

Number of bits/sample = $m = \log_2 L = \log_2 256 = 8$ bit

$$B_T = \frac{m \cdot f_s}{2} = \frac{8 \times 12000}{2} = 48kHz$$

Delta Modulation

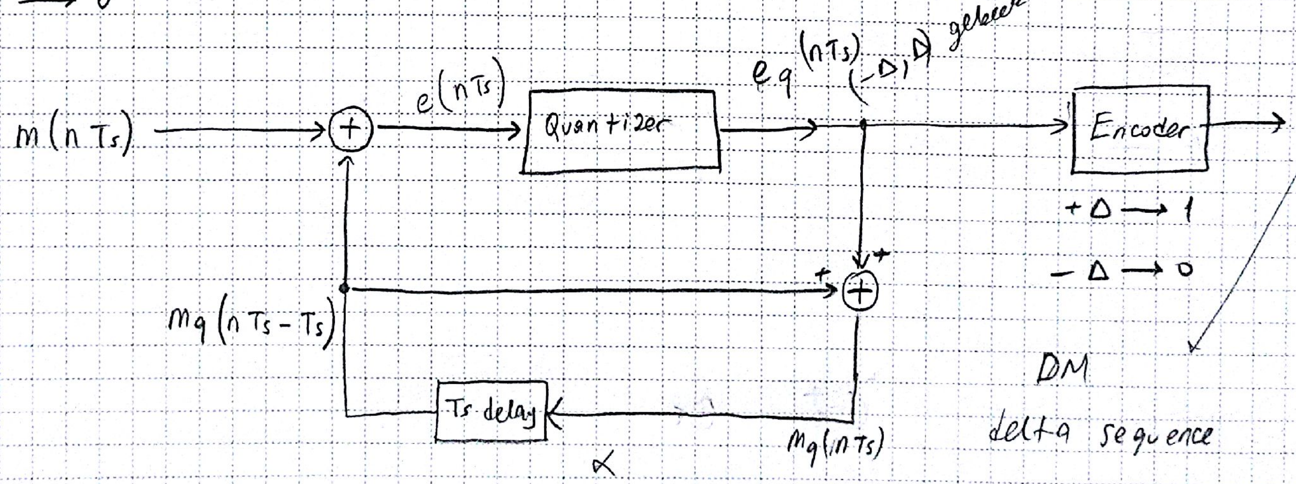


- basamaktan kurtarmak için Low pass'den geçirilmelidir.

1 1 1 0 1 0 0

+ Δ ⟶ 1

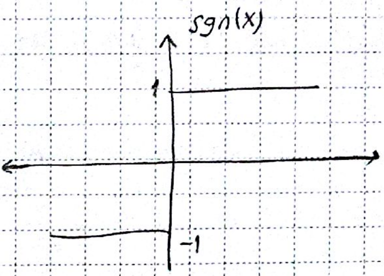
− Δ ⟶ 0



$$e(nT_s) = m(nT_s) - m_q(nT_s - T_s)$$

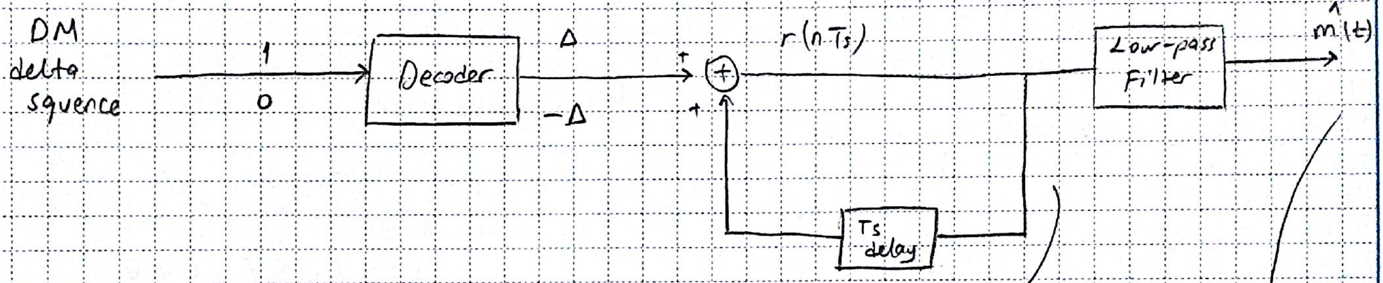
$$e_q(nT_s) = \Delta \, sgn[e(nT_s)]$$

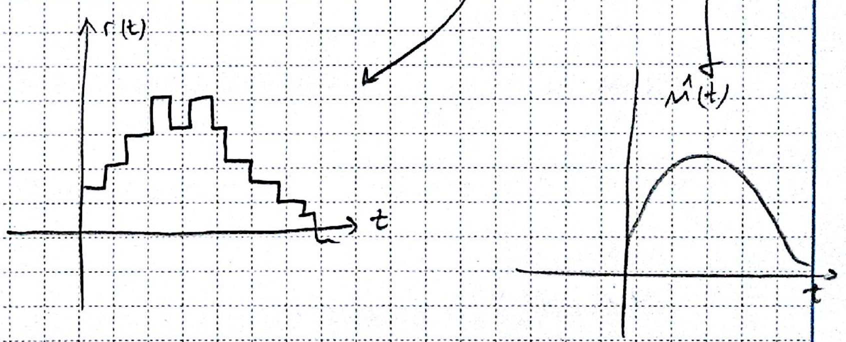
$$m_q(nT_s) = m_q(nT_s - T_s) + e_q(nT_s)$$



Receiver

1 1 0 0 1 1 0



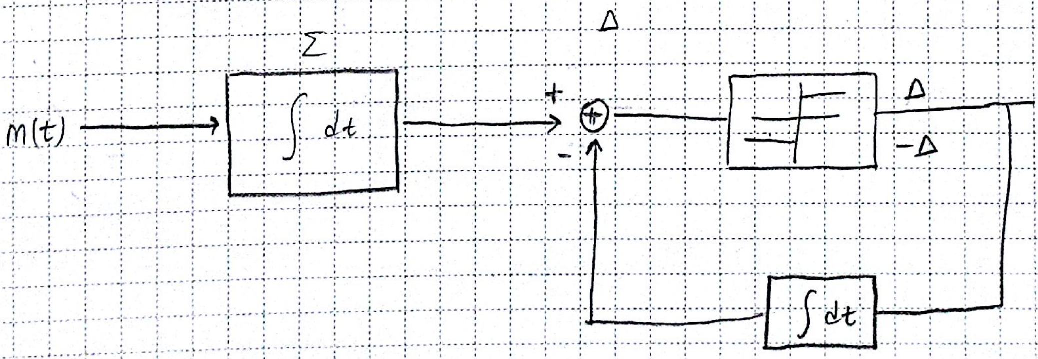


1 1 1 0 1 0 0

$\Delta \rightarrow 1$

$-\Delta \rightarrow 0$

Delta Sigma Modulation



Differantial Pulse Code Modulation (DPCM)

Taylor series expansion of quantized message signal

$$m_q(t+T_s) = m_q(t) + \frac{T_s}{1!} m'_q(t) + \frac{T_s}{2!}^{n} m_q^{n}(t) + - -$$

$$\approx m_q(t) + T_s\, m_q'(t)$$

$$m(kT_s) = m(k) \qquad m(kT_s \mp T_s) = m[k \mp 1]$$

$$m[k+1] \approx m(k) + T_s \left[\frac{m[k] - m[k-1]}{T_s} \right]$$

$$m[k+1] = 2\, m[k] - m[k-1]$$

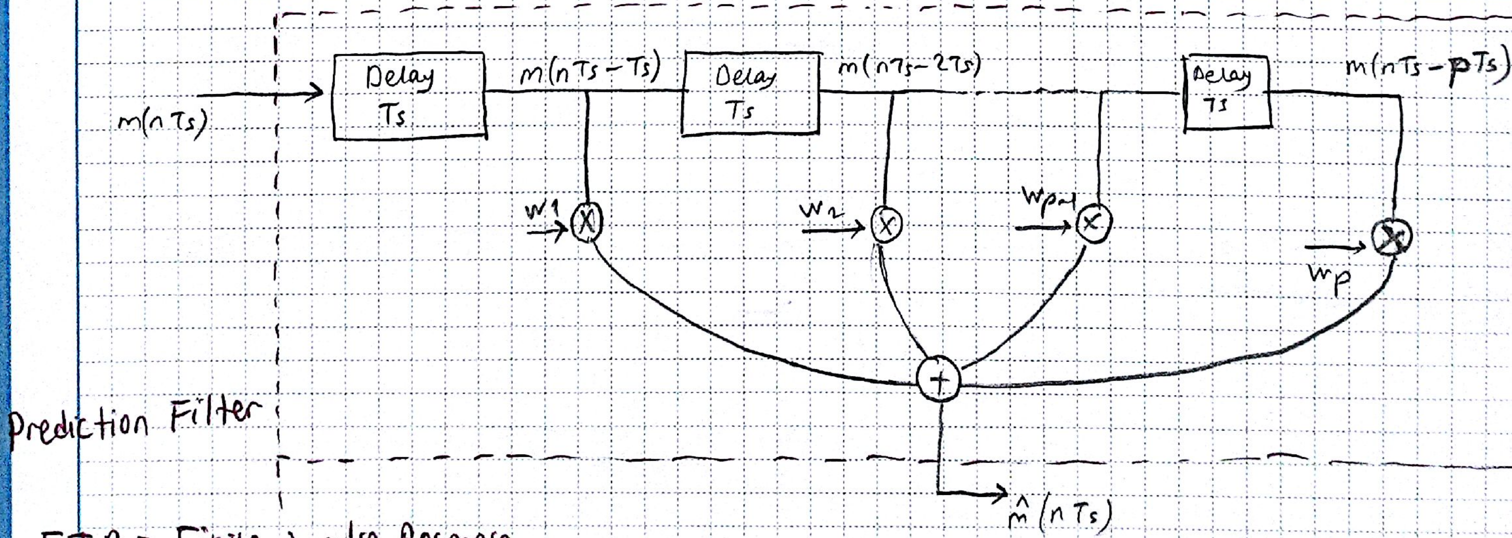
Crude prediction of (k+1) th sample from the two previous samples.

09/03/2016

DPCM

$$\hat{m}(nT_s) = w_1\, m(nT_s - T_s) + w_2\, m(nT_s - 2T_s) + - - - + w_p\, m(nT_s - pT_s)$$

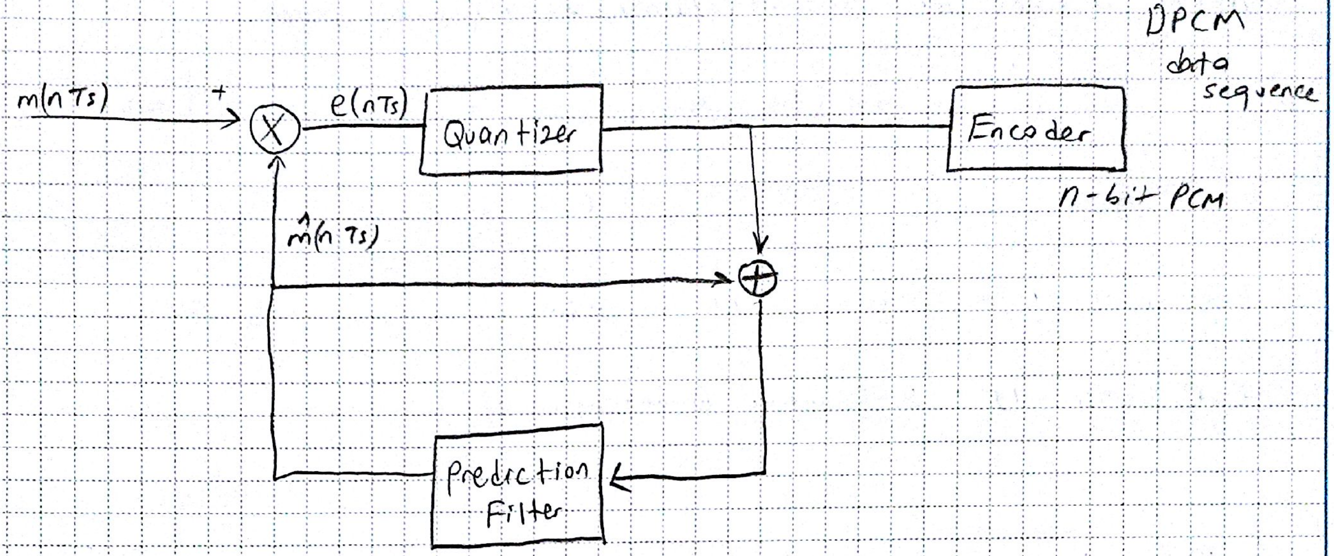
p^{th} order prediction



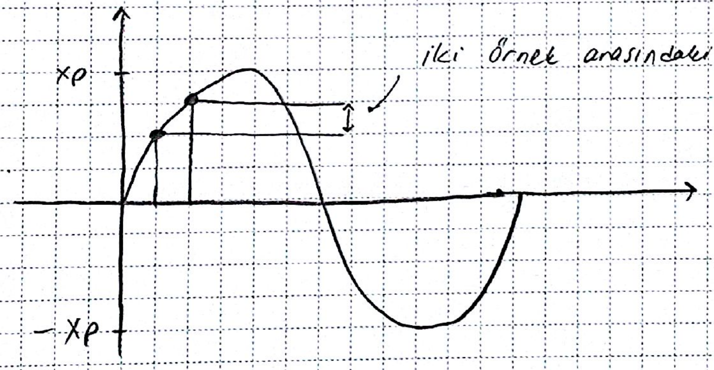
FIR = Finite impulse Response

Tarih: 09/03/2016

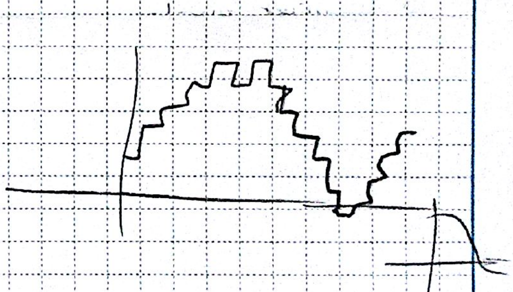
DPCM Transmitter



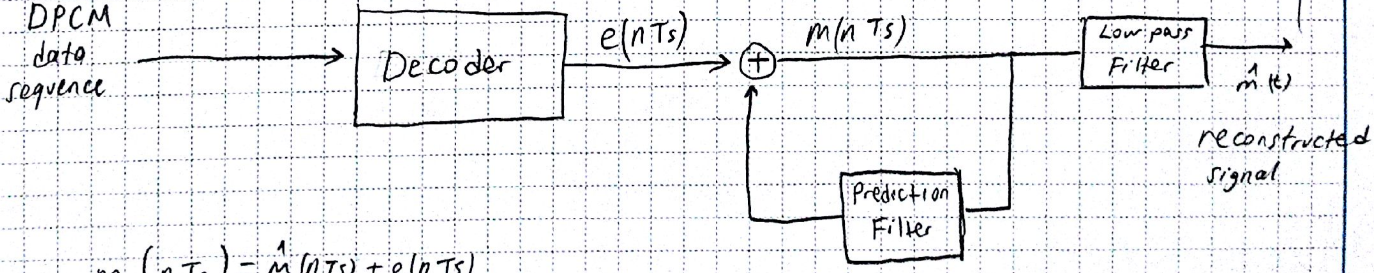
$$e(nT_s) = m(nT_s) - \hat{m}(nT_s)$$



iki örnek arasındaki farkı kodluyoruz.



DPCM Receiver

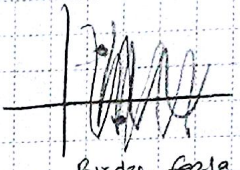


$$m(nT_s) = \hat{m}(nT_s) + e(nT_s)$$

DPCM vs DM

- DPCM includes delta Modulation (DM) as a special case

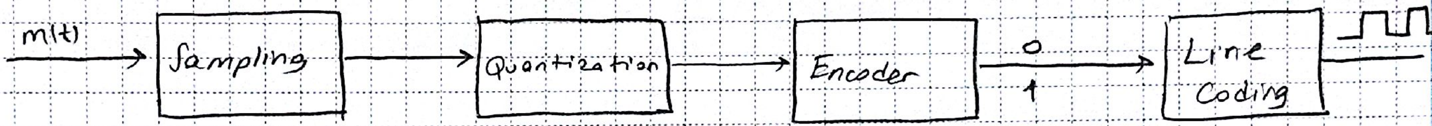
1) The use of a one bit (two level) quantizer in DM →



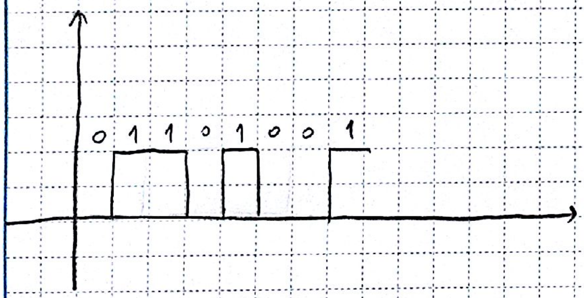
Birden fazla sayıyla kodlanabilir.

2) Replacement of the prediction filter in DPCM by a signal delay element.

C.C Cutler 1952 Differantial Quantization of Communication Signals



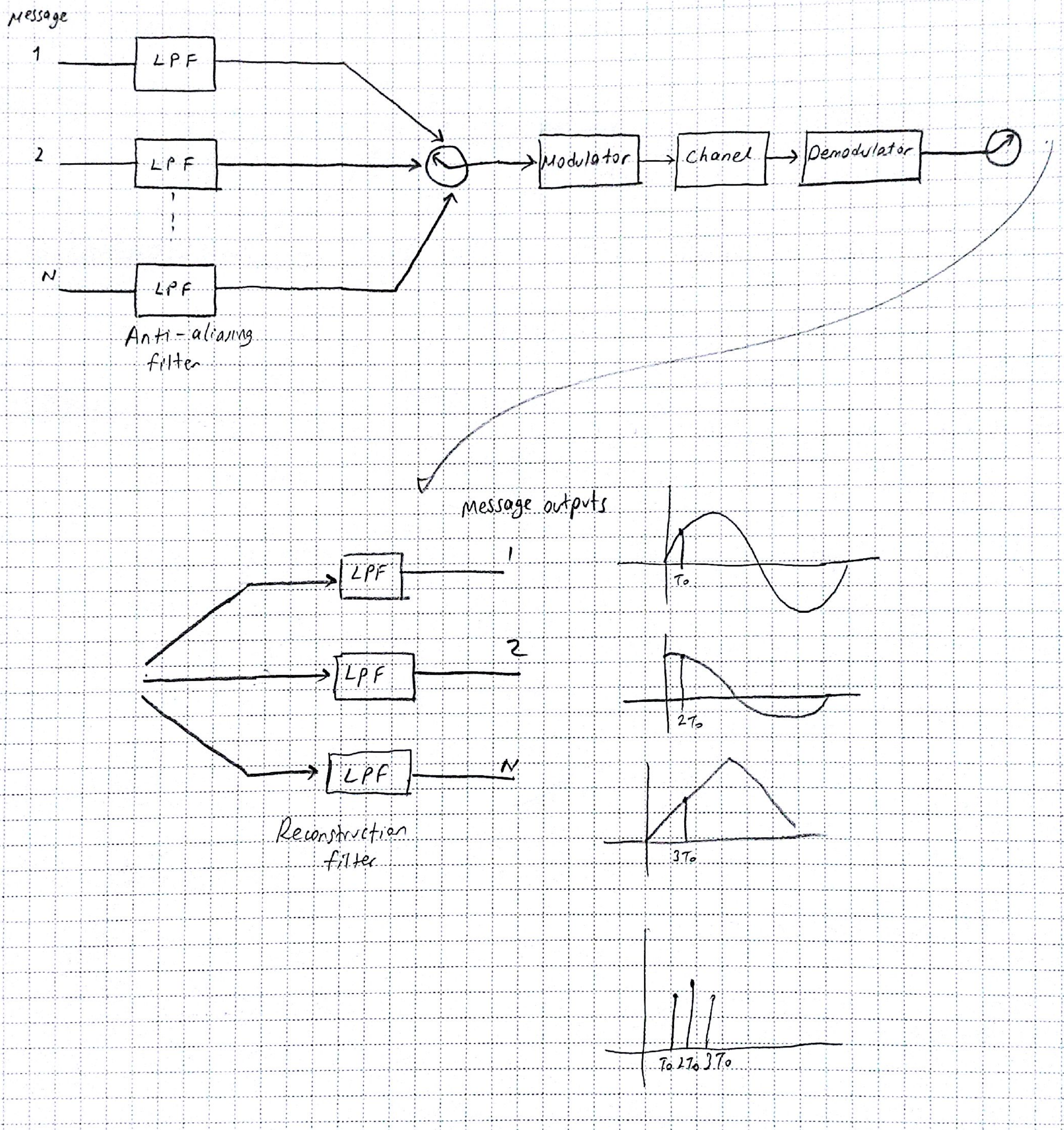
Line Codes



On-off signal (non-return to zero)

Tarih: 09/03/2016

Time Division Multiplexing (TDM)



α

Tarih: 15 / 03 / 2016

min <u>Band with (PCM)</u> : $B_{pcm} = \frac{m \cdot 2f_s}{2} = m \cdot f_s$

min <u>Bandwith (TDM)</u> : $B_{TDM} = N \cdot m \cdot f_s$

European System (E1)

- 30 voice channels +2 for frame sychronization and framing.

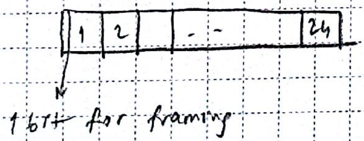
  32 channels
- 8 bit, sampling rate 8 kHz

$$R_{E1} = 32 \cdot 8 \cdot 8000 = 2.048 \text{ Mbps}$$

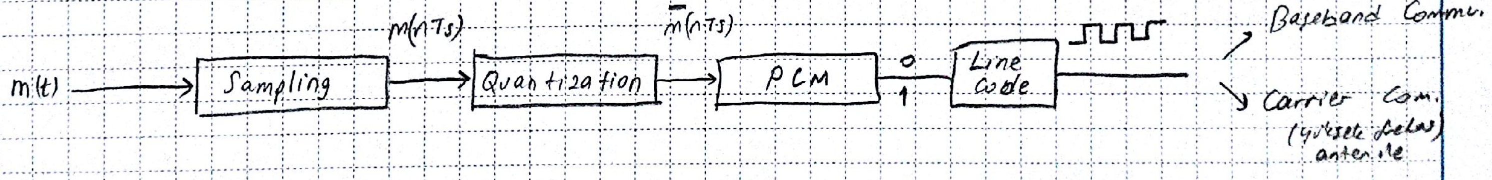
North America (T1)

- 24 voice channel + 1 single bit sychronization
- 8 bits, sampling rate 8 kHz

$$R_{T1} = [(24 \times 8) + \underset{\text{framing için kullanıldı}}{1}] \times 8000 = 1{,}544 \text{ Mbps}$$



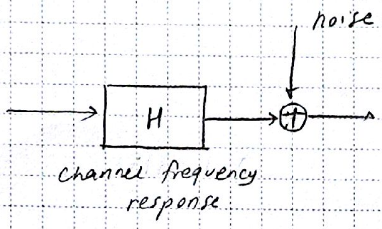
Line Code : Bitleri darbe dizilerine çeviriyor.

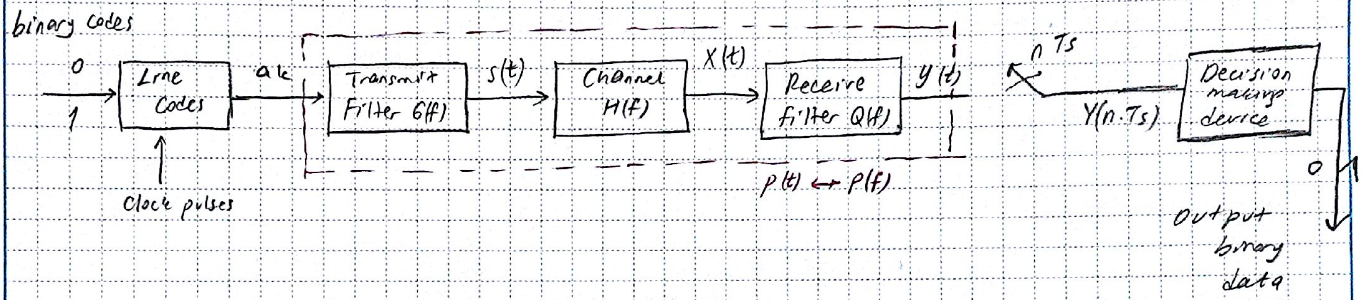


Baseband Comminition

1- Entersymbol Enterference (ESI) : which arises due to imperfections in the frequency response of the channel.

2- Channel noise





Binary bit stream : $b_k \in \{0, 1\}$

Encoded Signal : $a_k \begin{cases} +1 & b_k = 1 \\ -1 & b_k = 0 \end{cases}$

$$s(t) = \sum_{k=-\infty}^{\infty} a_k \cdot g(t - kT_s)$$

$$X(t) = s(t) \circledast h(t)$$

$$y(t) = X(t) \circledast q(t)$$

$$y(t) = s(t) \circledast h(t) \circledast q(t)$$

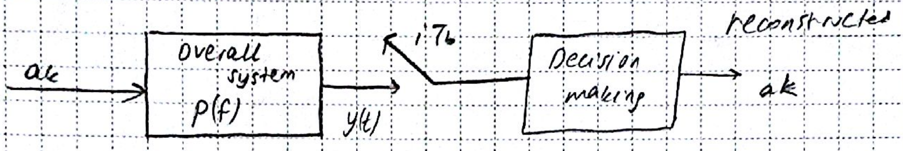
$$y(t) = \sum_{k=-\infty}^{\infty} a_k \cdot p(t - kT_s) \qquad p(t) = g(t) \circledast h(t) \circledast q(t)$$

sampled at $t = i\,T_b$

$$y(i\,T_b) = \sum_{k=-\infty}^{\infty} a_k \cdot p(iT_b - kT_b)$$

$y_i \triangleq y(iT_b)$ $p_i \triangleq p(iT_b)$

$$y_i = \sum_{k=-\infty}^{\infty} a_k \, p_{i-k}$$ Discrete Convolution



$P_u = P(0) = \sqrt{E}$ → Transmitted signal energy per bit

$$y_i = a_i\sqrt{E} + \sum_{\substack{k=-\infty \\ i\neq k}}^{\infty} a_k \, p_{i-k}$$

1. term represents the transmitted binary symbol except for the scaling factor $\sqrt{E}$

2. term is the combined effect of all other transmitted binary sysmbols intersymbol Interference (ISI)

In the absence of ISI and noise $y_i = a_i\sqrt{E}$

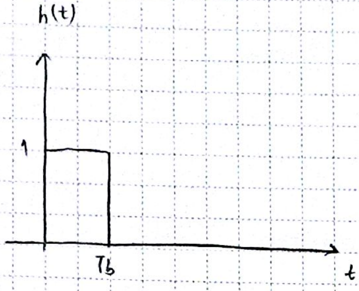
Pulse shaping Problems

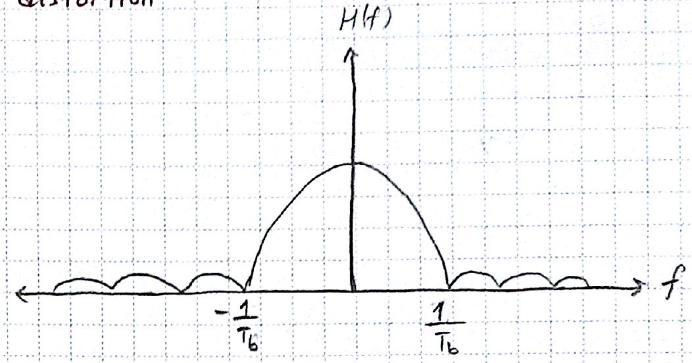
Given the channel transfer function $H(f)$, determine the transmit pulse spectrum $G(f)$ and receive filter transfer function $Q(f)$ so as to satisfy two basic requirements

1) ISI is reduced to zero
2) Transmission bandwith is conserved

∝

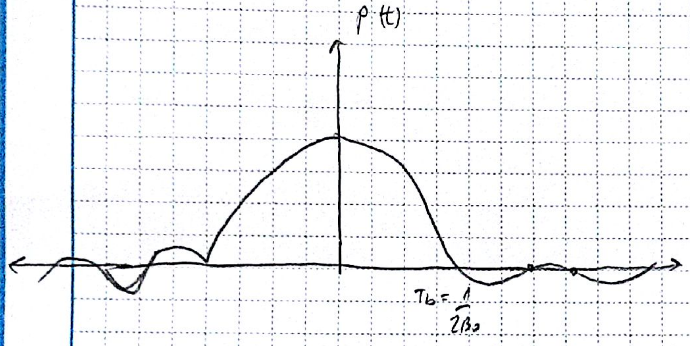
Time Limited pulses Spectral distortion

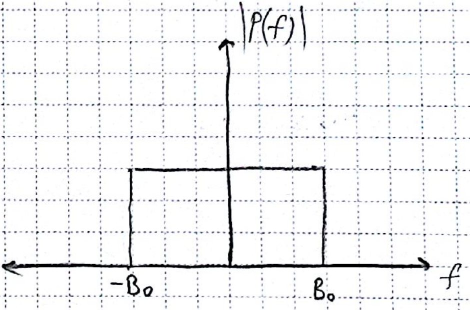




- Time-limited pulses cannot be band limited
- The part of their spectra is supressed by a band limited channel

Band Limited Pulses : ISI





- Band limited pulses can not be time limited
- Thus various pulses will overlap and cause ISI

Nyquist Criterion for zero ISI

For zero ISI

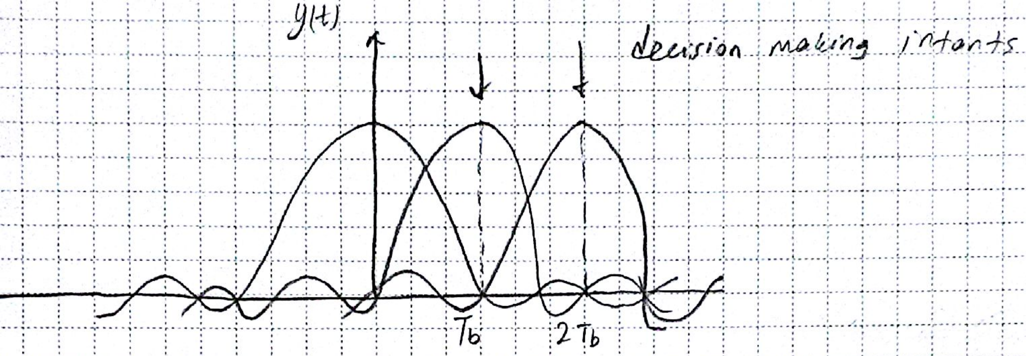
$$p_i = p(iT_b) = \begin{cases} \sqrt{E}, & i=0 \\ 0, & i=1,2,3 \end{cases}$$

The optimum pulse is sinc

$$p_{opt}(t) = \sqrt{E}\,\mathrm{sinc}(2B_0 t) \qquad t = i\,\frac{1}{2B_0}$$

Assume that $p(t)$ is band limitted to $-B_0 < f < B_0$

$$y(t) = \sum_{i=-\infty}^{\infty} p\left(\frac{i}{2B_0}\right) \text{sinc}(2B_0 t - i)$$



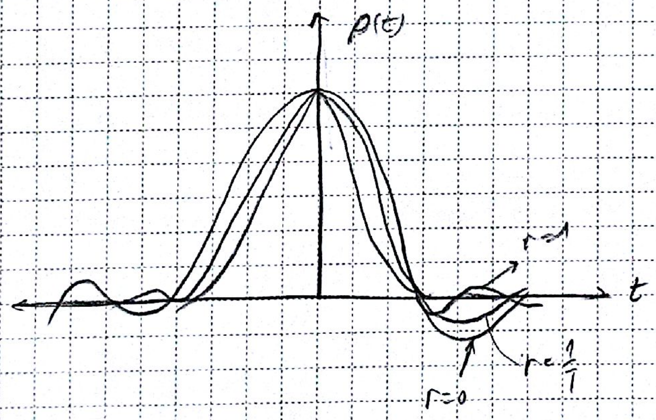
- Pulse amplitude can be detected correctly despite pulse overlapping If there is no ISI at the decision making instants $T_b, 2T_b, ---$
- Sinc pulse is infinite length
- It decays too slowly $1/t$

Raised - Cosine Pulse

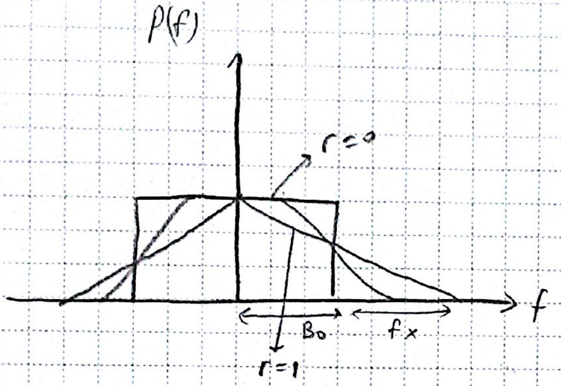
Faster decaying

$$p(t) = \sqrt{E} \quad \text{sinc}(2B_0 t) \left[\frac{\cos(2\pi r B_0 t)}{1 - 16 r^2 B_0^2 t^2} \right]$$

$$0 \leq r \leq 1$$
$\downarrow$
roll of factor

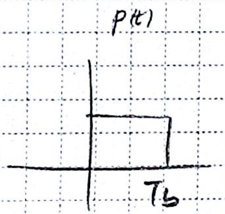


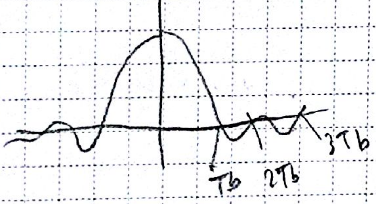
$$P(f) = \begin{cases} \frac{\sqrt{E}}{2B_0}, & 0 \leq |f| \leq f_1 \\ \frac{\sqrt{E}}{4B_0}\left\{1 + \cos\left[\frac{\pi(|f|-f_1)}{2(B_0-f_1)}\right]\right\}, & f_1 \leq |f| \leq 2B_0 - f_1 \\ 0, & 2B - f \leq |f| \end{cases}$$

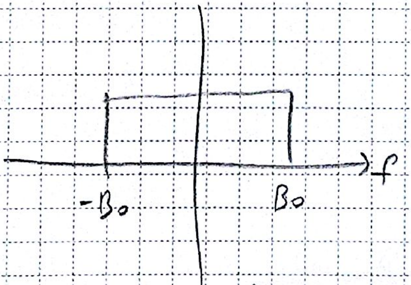


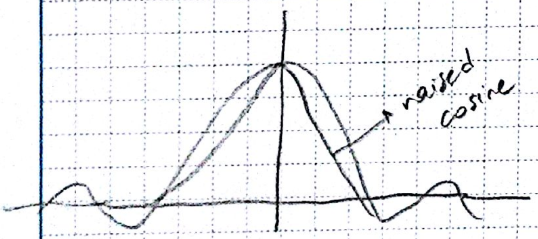
$$\text{Roll-off Factor} = \frac{\text{excess bandwith}}{\text{therolted min bandwith}} = \frac{f_x}{B_0}$$

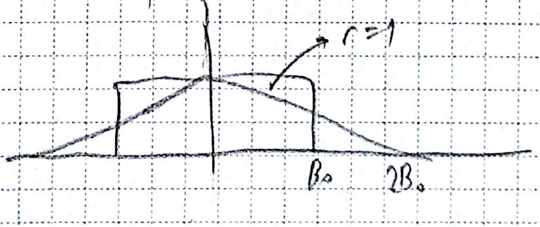
16. 03. 2016







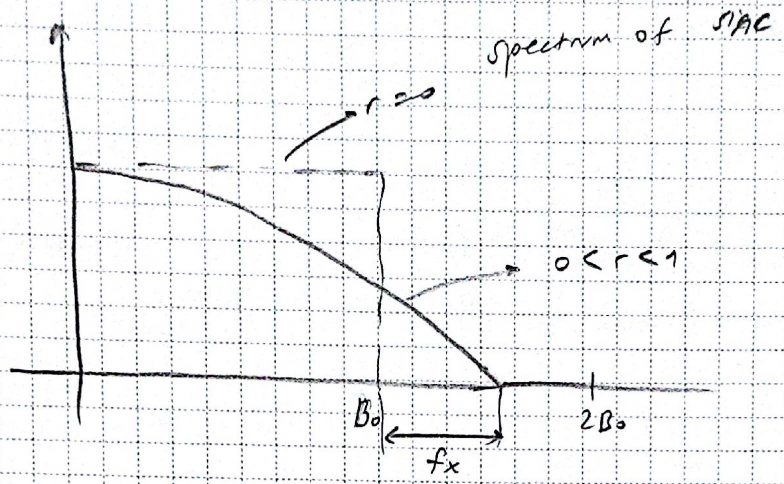




Passiflora moonlight Costa Rica
—Seda—

Tarih: 15 / 03 /2016

Roll of Factor



$r = \frac{f_x}{B_0} \Rightarrow f_x = r \cdot B_0 \qquad 0 \leq r < 1$

sampling rate

minimum $B_T = \frac{f_s}{2} = \frac{2(B_0 + f_x)}{2} = B_0 + r \cdot B_0 = (1+r) B_0$

$$B_T = (1+r) B_0$$

Example =

The signal bandlimited to $B = 4kHz$ is sampled at Nyquist rate. The obtained signal is transmitted through

a) A Nyquist channel

b) A channel with raised-cosine chracteristics with roll-off factor $r=1$

Calculate the required channel bandwith for each case.

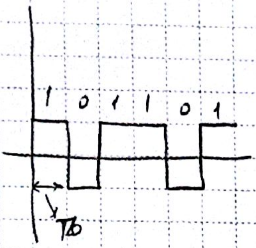
Nyquist sample rate $f_s = 2B = 2 \cdot 4000 = 8kHz$

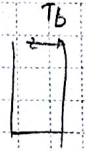
a) $B_T = \frac{f_s}{2} = \frac{8000}{2} = 4000 = 4kHz$

b) $B_T = B \cdot (1+r)$ $\qquad f_s = 2B(1+r)$

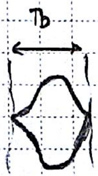
$= 4000 \cdot (1+1) = 8kHz$

Eye Pattern

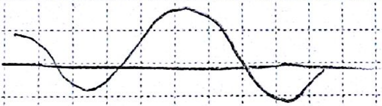


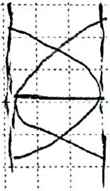


Ideal Nyqvist Channel

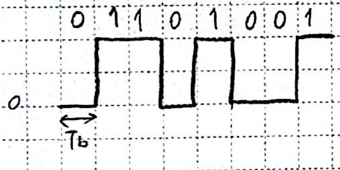


Zero ISI

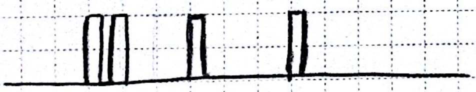




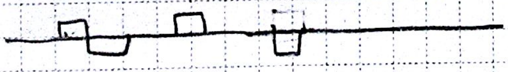
with ISI



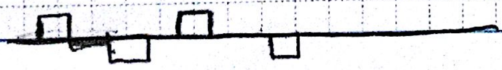
on-off Ntr



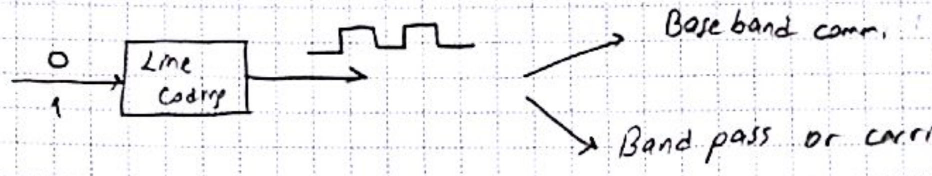
on Off Rz



Bipolar NRZ



Bipolar Rz



Band-pass Communication

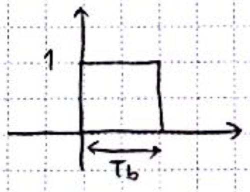
Binary Amplitude - Shift Keying (BASK)
" Frequency " " (BFSK)
" Phase " " (BPSK)

Quadrature Phase-Shift Keying (QPSK)

22/03/2016

Binary Amplitude Shift Keying (BASK)

rrier $C(t) = \sqrt{\frac{1}{T_b}} \cos(2\pi f_c t + \theta_c)$

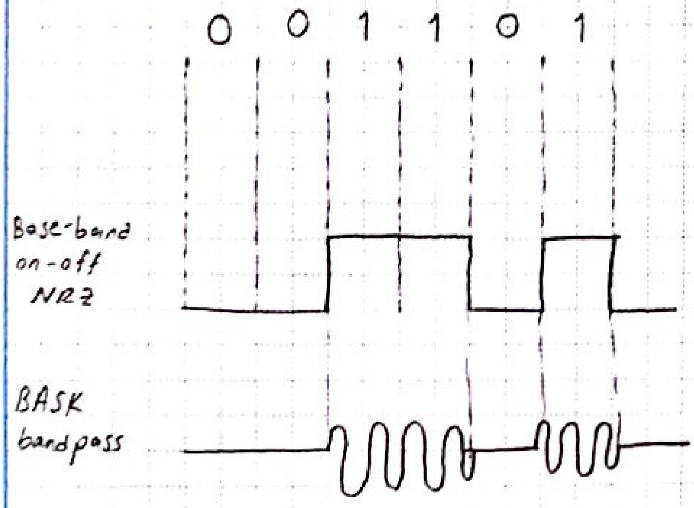


Binary data system obtained by on-off signaling.

$$b(t) = \begin{cases} \sqrt{E_b} & , \text{binary signal } 1 \\ 0 & , \text{binary signal } 0 \end{cases}$$

BASK wave

$$s(t) = \begin{cases} \sqrt{\frac{E_t}{T_b}} \cos(2\pi f_c t) & , \text{for } 1 \\ 0 , & \text{for } 0 \end{cases}$$

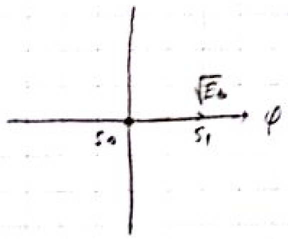


Signal space representation of BASK (Signal constellation)

$$s_1(t) = \sqrt{\frac{E_b}{T_b}} \cos(2\pi f_c t)$$

$$s_0(t) = 0$$

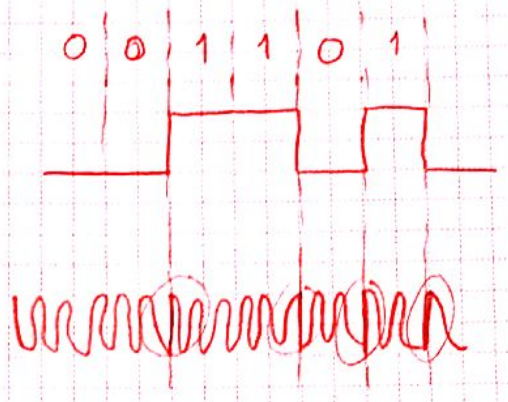
$$\varphi(t) = \sqrt{\frac{1}{T_b}} \cos(2\pi f_c t) \quad \text{basis function}$$



Binary Phase-Shift Keying (BPSK)

$$c(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \theta_c) \quad : \text{carrier}$$

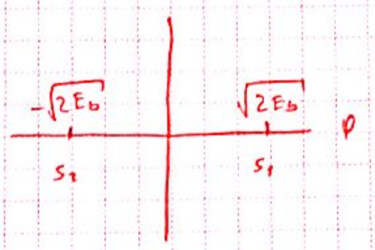
$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & , \text{binary sysmbol } 1,\ i=1 \\ \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t) & \text{binary sysmbol } 0,\ i=2 \end{cases}$$



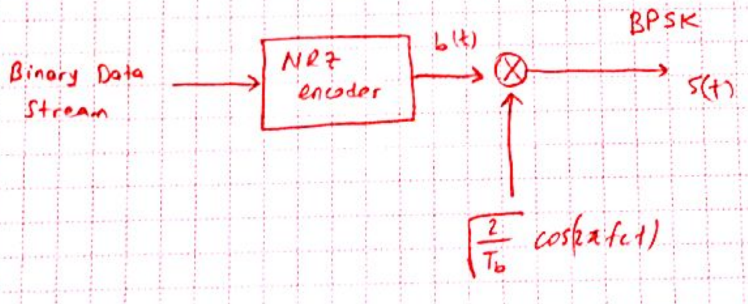
$s_1(t) = \sqrt{\frac{E_b}{T_b}} \cos(2\pi f_c t)$

$\varphi(t) = \sqrt{\frac{1}{T_b}} \cos(2\pi f_c t)$

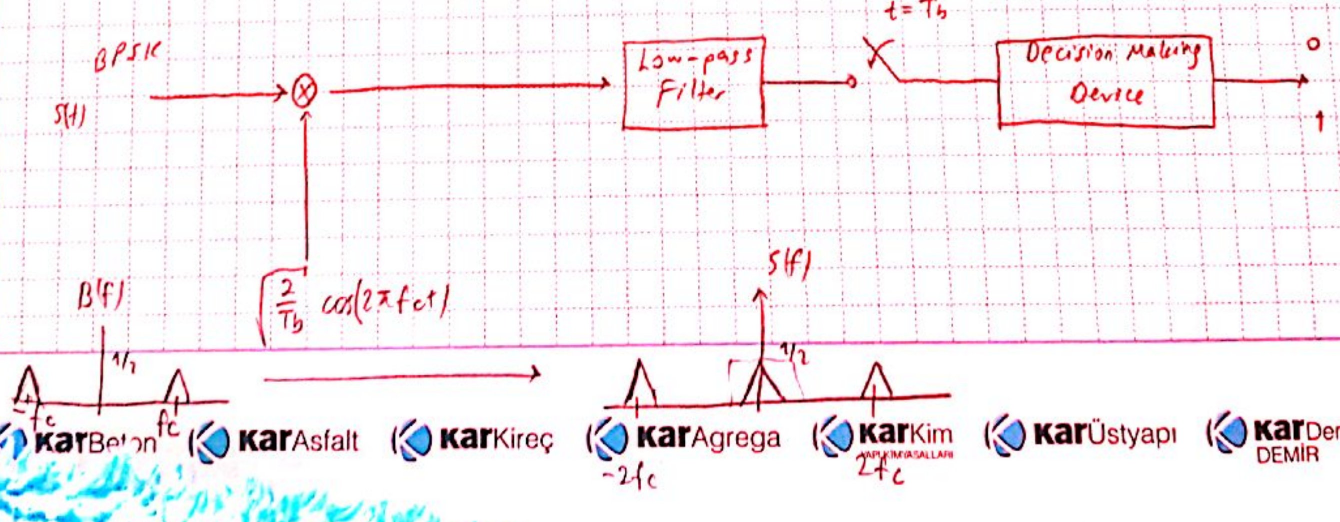
$s_2(t) = -\sqrt{\frac{E_b}{T_b}} \cos(2\pi f_c t)$



Generation and Coherent Detection of BPSK signals



Receiver



Quadri Phase Shift Keying (QPSK)

$$S_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + (2i-1) \cdot \frac{\pi}{4}) , & 0 \leq t \leq 2T_b \\ 0 , & \text{otherwise} \end{cases}$$

dibit

$\{10, 00, 01, 11\}$

$$S_i(t) = \sqrt{\frac{2E_b}{2T_b}} \cos\left((2i-1)\frac{\pi}{4}\right) \cos(2\pi f_c t)$$

$$- \sqrt{\frac{2E_b}{2T_b}} \sin\left((2i-1)\frac{\pi}{4}\right) \sin(2\pi f_c t)$$

$$a_1(t) = \sqrt{E_b} \cos\left((2i-1)\frac{\pi}{4}\right) = \begin{cases} \sqrt{E_b/2} , & i=1,4 \\ -\sqrt{E_b/2} , & i=2,3 \end{cases}$$

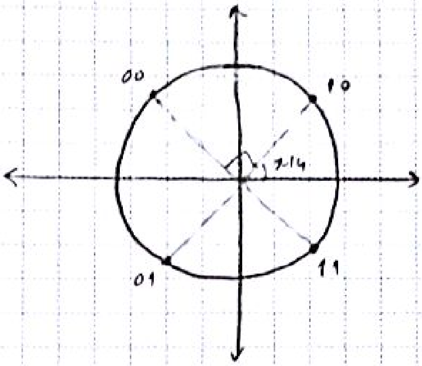
$$a_2(t) = \sqrt{E_b} \sin\left((2i+1)\frac{\pi}{4}\right) = \begin{cases} -\sqrt{E_b/2} , & i=1,2 \\ +\sqrt{E_b/2} & i=2,3 \end{cases}$$

i	Phase of QPSK	Binary wave 1 $a_1(t)$	Binary wave 2 $a_2(t)$	input bits $0 \leq t \leq 2T_b$
1 -	$\frac{\pi}{4}$	$\sqrt{E_b/2}$	$-\sqrt{E_b/2}$	1 0
2 -	$\frac{3\pi}{4}$	$-\sqrt{E_b/2}$	$-\sqrt{E_b/2}$	0 0
3 -	$\frac{5\pi}{4}$	$-\sqrt{E_b/2}$	$\sqrt{E_b/2}$	0 1
4 -	$\frac{7\pi}{4}$	$\sqrt{E_b/2}$	$\sqrt{E_b/2}$	1 1

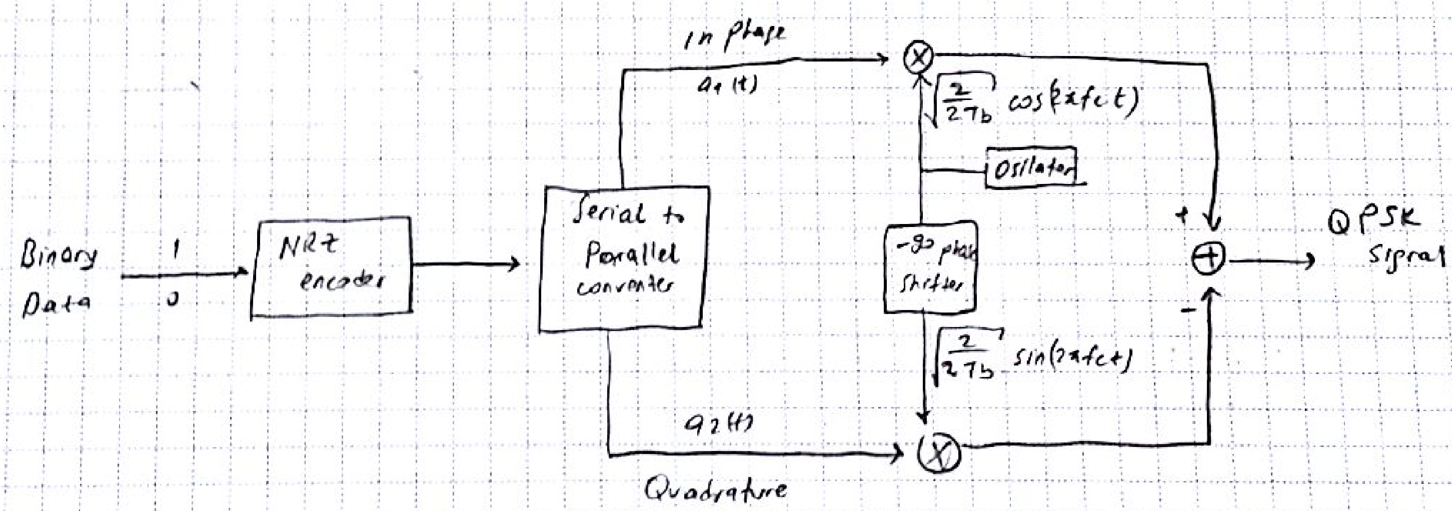
Signal Space Representation of QPSK

$$S_1(t) = \sqrt{\frac{2E_b}{2T_b}} \cos\left((2i-1)\frac{\pi}{4}\right) \cos(2\pi f_c t) - \sqrt{\frac{2E_b}{2T_b}} \sin\left((2i+1)\frac{\pi}{4}\right) \sin 2\pi f_c t$$

$$\varphi_1(t) = \sqrt{\frac{2}{2T_b}} \cos(2\pi f_c t) \qquad \varphi_2(t) = \sqrt{\frac{2}{2T_b}} \sin(2\pi f_c t)$$



Generation of QPSK



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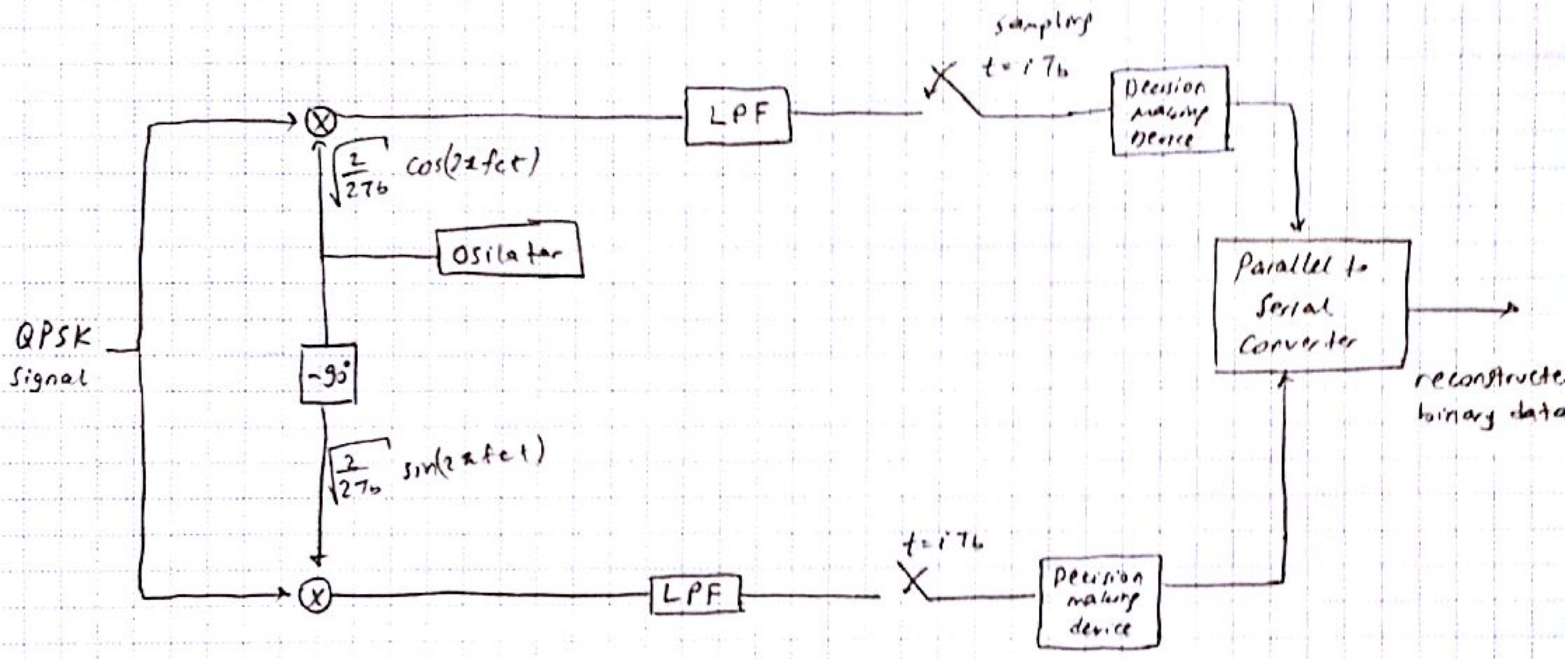
Coherent Dedection of QPSK



QPSK Signal